10|10|23

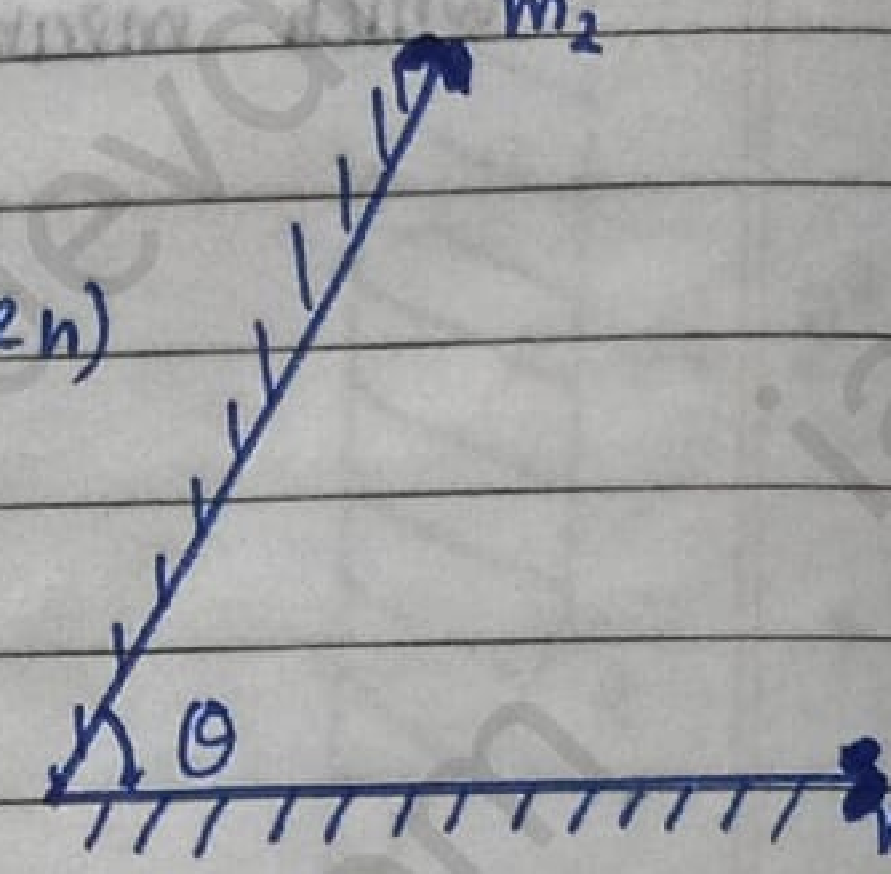
CHAPTER – 9
RAY OPTICS

★ CONCEPT OF INCLINED PLANE

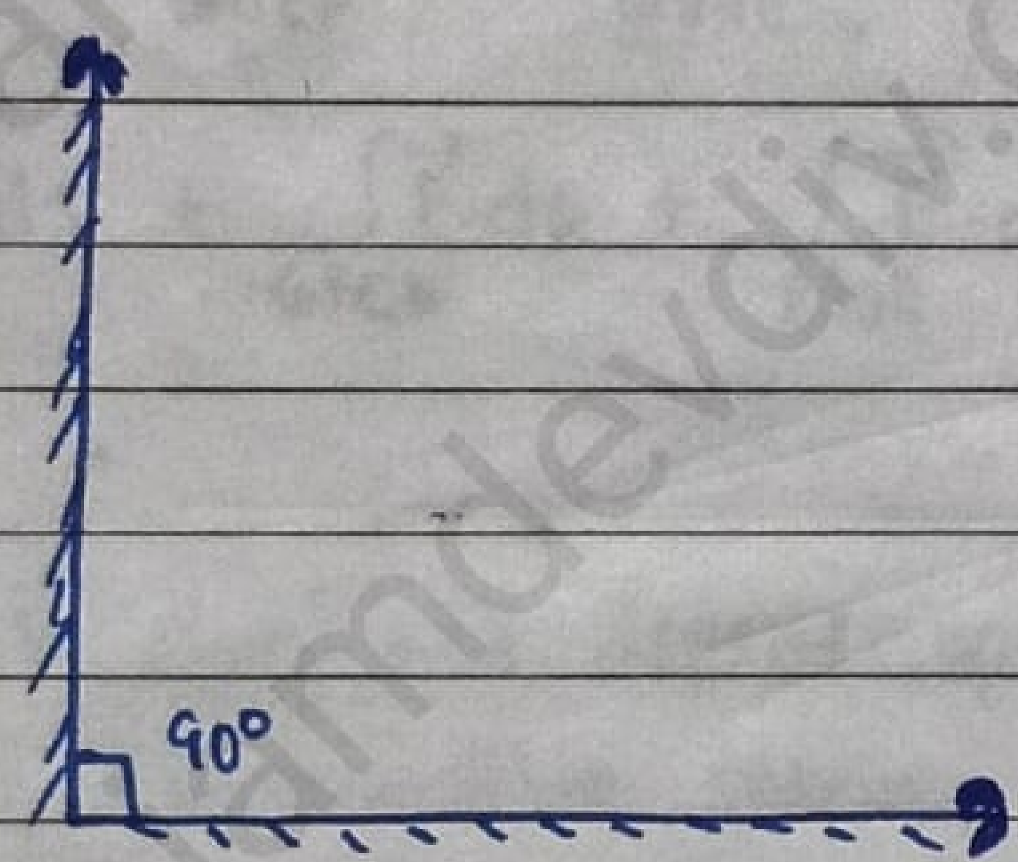
~~Examples~~

$n = \frac{360^\circ}{\theta}$ (odd) (OR) $n = \frac{360^\circ}{\theta} - 1$ (even)

↳ number of images

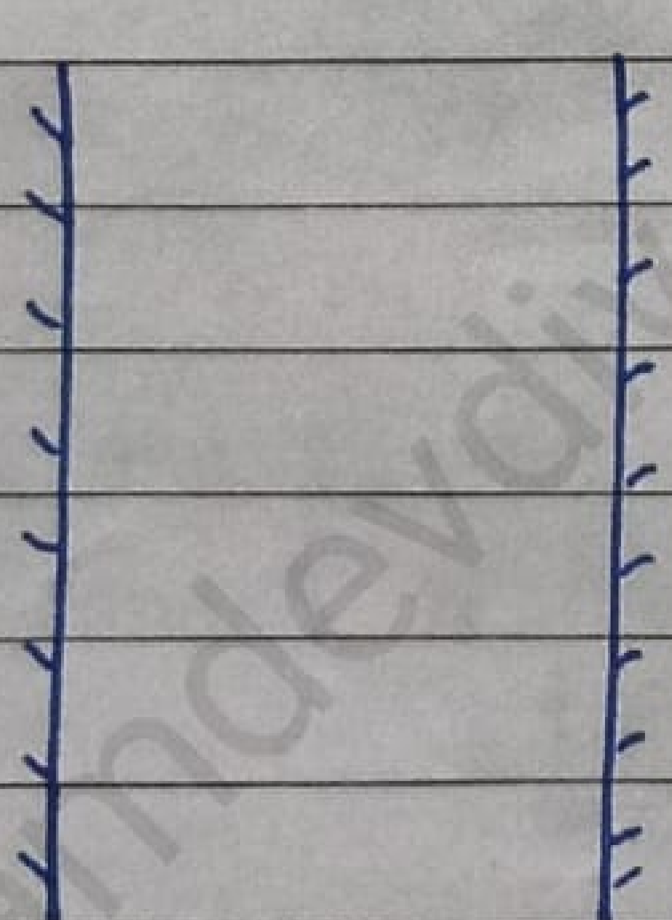


EXAMPLES:



$n = \frac{360^\circ}{90^\circ} = 4$ (even)

$n = 4 - 1 = 3$



$n = \frac{360^\circ}{0} = \infty$

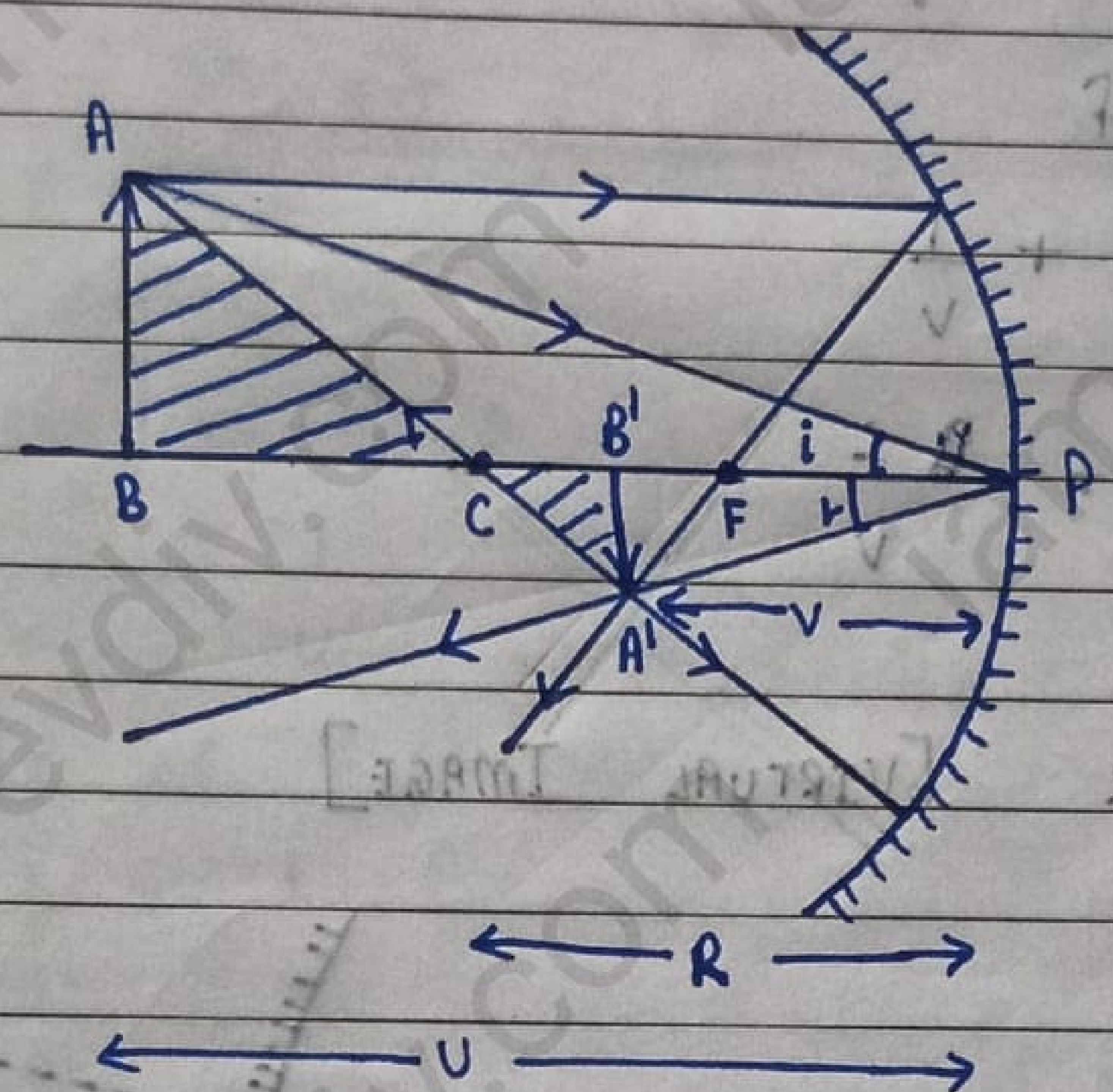
★ PROPERTIES OF PLANE MIRROR

(i) Image formed is virtual and erect, and at the same distance behind the mirror.

(ii) Always formes lateral inversion.

★ MIRROR FORMULAS

• CONCAVE MIRROR [REAL IMAGE]



ΔABC and $\Delta A'B'C$ are similar

$$\frac{AB}{A'B'} = \frac{BC}{B'C} \quad \text{--- (1)}$$

ΔABP and $\Delta A'PB'$

$$\frac{AB}{A'B'} = \frac{BP}{B'P} \quad \text{--- (2)}$$

Comparing (1) and (2),

$$\frac{BP}{B'P} = \frac{BC}{B'C} = \frac{BP - PC}{PC - B'P}$$

$$\Rightarrow \frac{-u}{-v} = \frac{-u-(-R)}{(-R)-(-v)}$$

$$\Rightarrow \frac{u}{v} = \frac{R-u}{v-R}$$

$$\Rightarrow u(v-R) = v(R-u)$$

$$\Rightarrow uv - uR = vR - uv$$

$\Rightarrow 2uv = vR + uR$

Dividing by uvR,

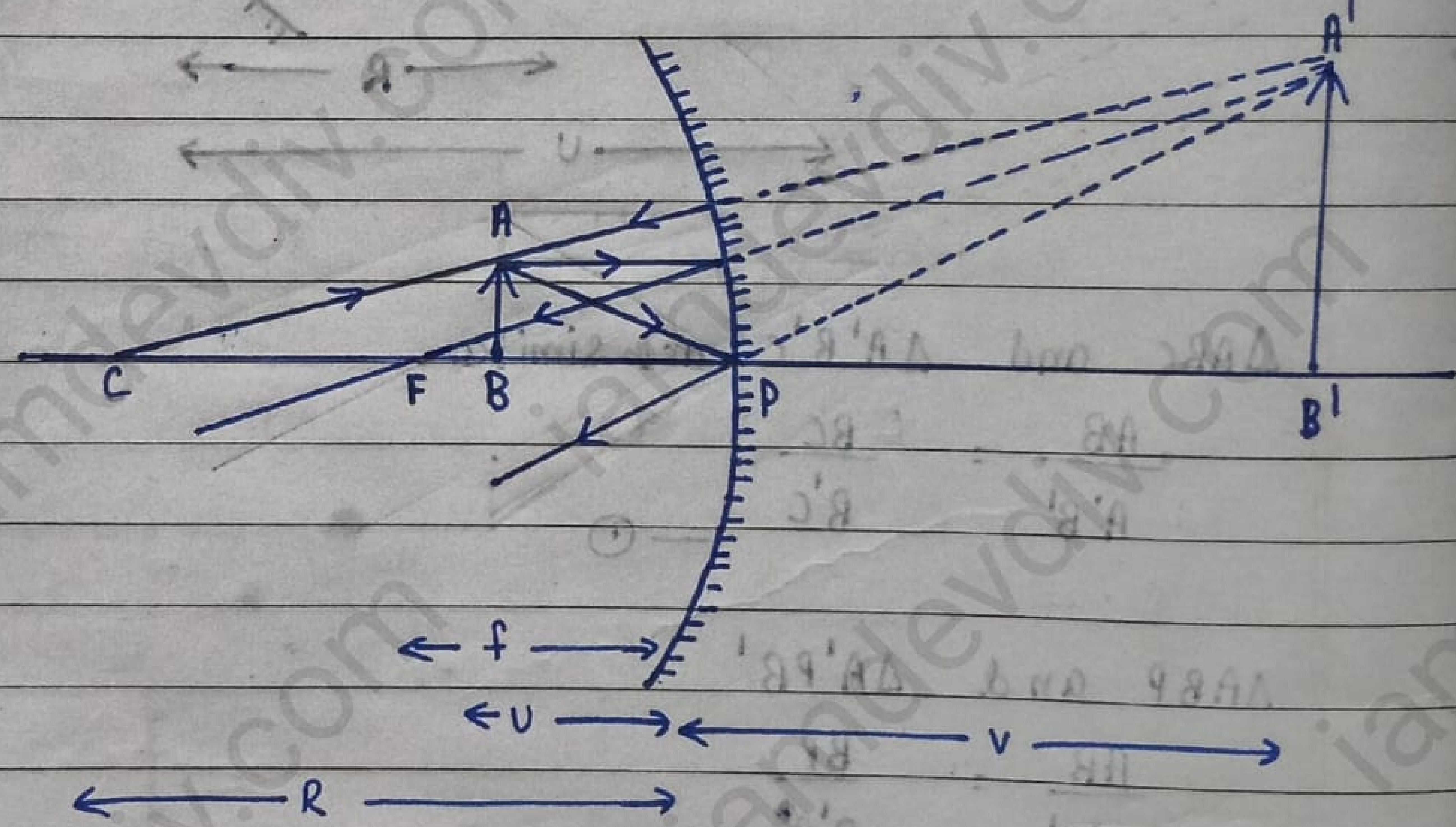
$$\Rightarrow \frac{2}{R} = \frac{1}{u} + \frac{1}{v}$$

$\because R = 2f$

$$\Rightarrow \frac{2}{2f} = \frac{1}{u} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

- CONCAVE MIRROR [VIRTUAL IMAGE]



ΔABC and $\Delta A'B'C$ are similar

$$\frac{AB}{A'B'} = \frac{CB}{B'C} \quad \text{—— (1)}$$

ΔABP and $\Delta A'B'P$ are similar

$$\frac{AB}{A'B'} = \frac{BP}{B'P} \quad \text{—— (2)}$$

From (1) and (2),

$$\frac{BP}{B'P} = \frac{CB}{B'C} = \frac{PC - PB}{CP + PB'}$$

$$\Rightarrow \frac{-u}{v} = \frac{-R-(-u)}{-R+v}$$

$$\Rightarrow -u(v-R) = v(u-R)$$

$$\Rightarrow -uv + uR = uv - vR$$

$$\Rightarrow 2uv = uR + vR$$

- Dividing by uvR

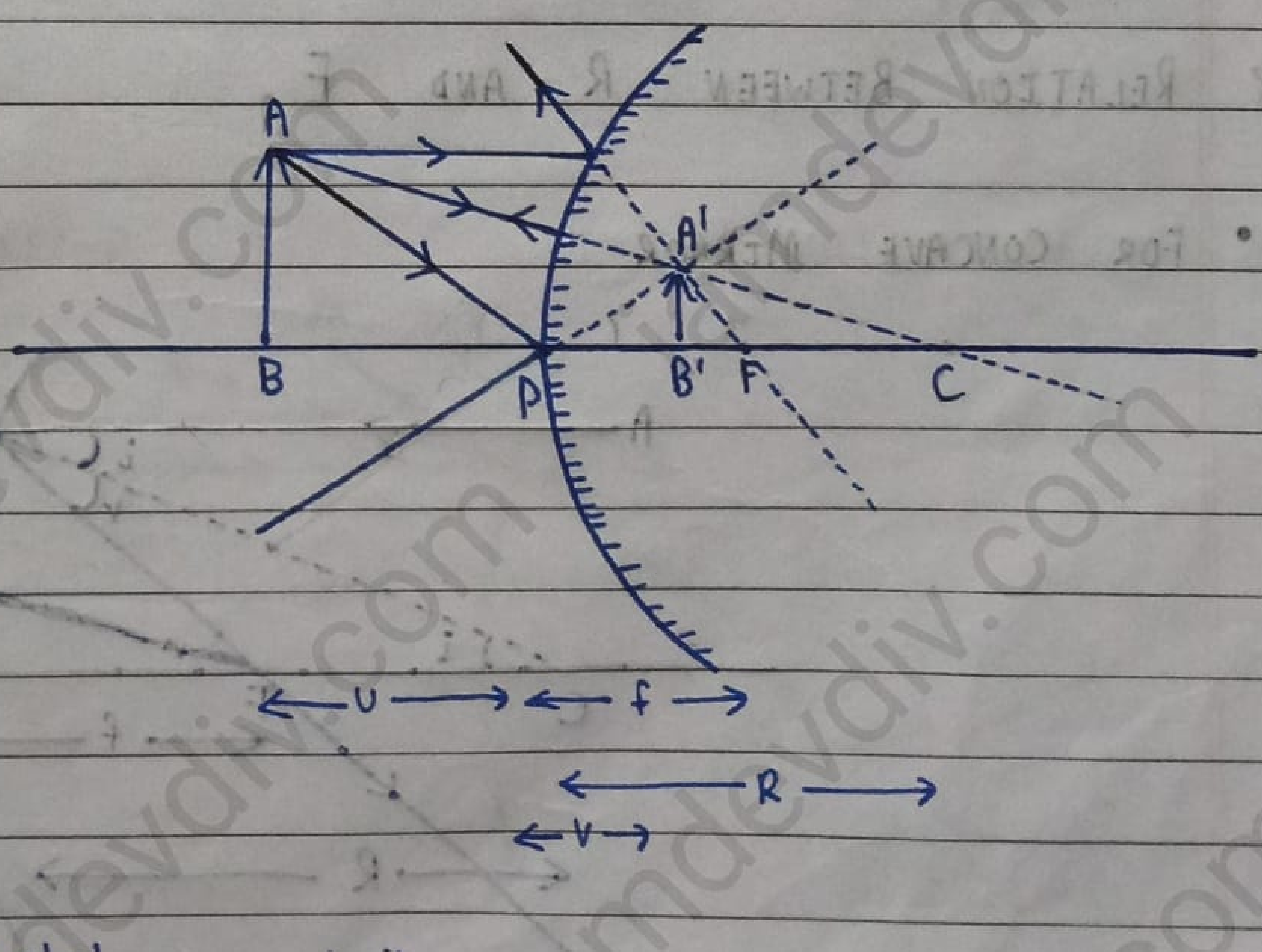
$$\Rightarrow \frac{2}{R} = \frac{1}{v} + \frac{1}{u}$$

$$\because R = 2f$$

$$\Rightarrow \frac{2}{2f} = \frac{1}{u} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

- CONVEX MIRROR [VIRTUAL IMAGE]



ΔABC and $\Delta A'B'C$ are similar,

$$\frac{AB}{A'B'} = \frac{BC}{B'C} \quad (1)$$

ΔABP and $\Delta A'B'P$ are similar,

$$\frac{AB}{A'B'} = \frac{BP}{PB'} \quad \text{— (2)}$$

From (1) and (2),

$$\frac{BP}{PB'} = \frac{BC}{B'C} = \frac{BP + PC}{PC - PB'}$$

$$\Rightarrow \frac{-u}{v} = \frac{-u + R}{R - v}$$

$$\Rightarrow -u(R - v) = v(R - u)$$

$$\Rightarrow -uR + uv = vR - uv$$

$$\Rightarrow 2uv = vR + uR$$

Dividing by uvR

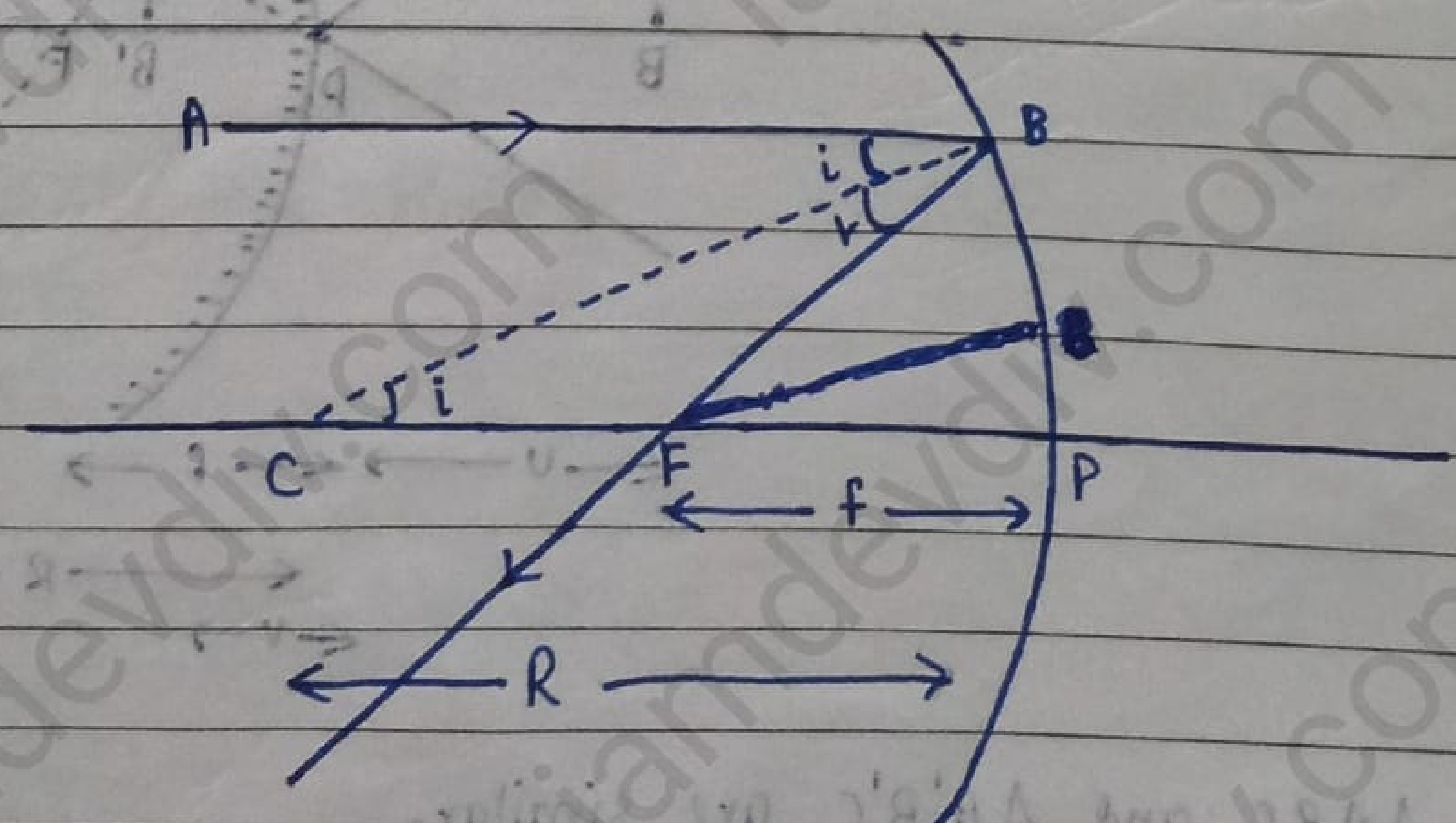
$$\Rightarrow \frac{2}{R} = \frac{1}{u} + \frac{1}{v}$$

$$\because R = 2f$$

$$\Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

★ RELATION BETWEEN R AND F

- FOR CONCAVE MIRROR



$\angle i = \angle r$ (by law of reflection)
$\angle i = \angle i$ (Alternate interior angles)

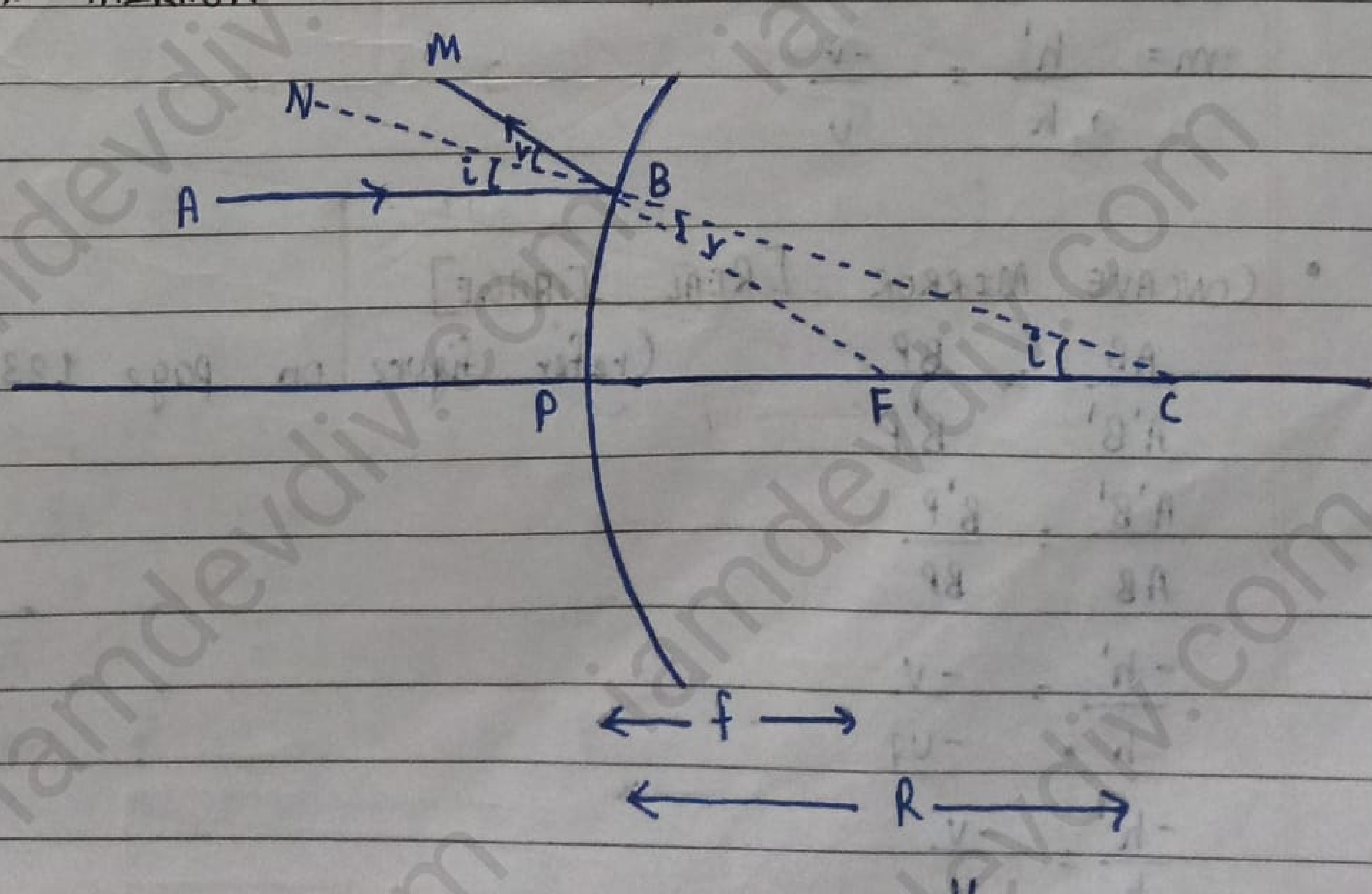
In ΔBCF,
$\angle i = \angle r$
$FB = FC$ — ①
i.e. ΔBCF is an isosceles triangle

For small aperture,
it means B lies near to P
$FP = FB$ — ②

From ① and ②,
$FP = FC$

$CP = CF + FP$
$\Rightarrow CP = FP + FP$
$\Rightarrow R = f + f$
$\Rightarrow R = 2f$

- FOR CONVEX MIRROR



$\angle i = \angle r$ (law of reflection)
$\angle NBM = \angle FBC = \angle r$ (vertically opposite angles)
$\angle ABN = \angle PCB = \angle i$ (corresponding angles)

In ΔBCF,
$\angle i = \angle r$
$FB = FC$ — ①
i.e. ΔBCF is an isosceles triangle

For small aperture, it means B lies near to P
$FP = FB$ — ②

From ① and ②,
$FP = FC$

$CP = CF + FP$
$\Rightarrow CP = FP + FP$
$\Rightarrow R = f + f$
$\Rightarrow R = 2f$

★ MAGNIFICATION FORMULA

$$m = \frac{h'}{h} = \frac{-v}{u}$$

- CONCAVE MIRROR [REAL IMAGE]

$$\frac{AB}{A'B'} = \frac{BP}{B'P}$$ (refer figure on page 123)

$$\frac{A'B'}{AB} = \frac{B'P}{BP}$$

$$\frac{-h'}{h} = \frac{-v}{-u}$$

$$\frac{-h'}{h} = \frac{-v}{u}$$

- CONCAVE MIRROR [VIRTUAL IMAGE]

$\frac{A'B'}{AB} = \frac{PB'}{PB}$ (refer figure on page 124)

$\frac{h'}{h} = \frac{v}{-u}$

$\frac{h'}{h} = \frac{-v}{u}$

- CONVEX MIRROR [VIRTUAL IMAGE]

$\frac{h'}{h} = \frac{-v}{u}$

* PROPERTIES OF MAGNIFICATION

$m \Rightarrow +ve \Rightarrow$ image is virtual, erect

$m > 1 \Rightarrow$ image is enlarged

$m < 1 \Rightarrow$ image is dimid

Q An object is placed 18 cm in front of mirror. If the image is formed at 4cm to the right of the mirror, calculate its focal length. What is the nature and radius of curvature.

Sol. $u = -18$

$v = 4cm$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\Rightarrow \frac{1}{f} = \frac{1}{-18} + \frac{1}{4}$

$\Rightarrow \frac{1}{f} = \frac{-2+9}{36}$

$\Rightarrow \frac{1}{f} = \frac{7}{36}$

$\Rightarrow f = \frac{36}{7}$ cm

$R = 2f$

$R = 2 \times \frac{36}{7} = \frac{72}{7}$ cm

Nature = Convex mirror

Q A 4.5cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of image and magnification.

Sol. $u = -12$ cm

$f = 15$ cm

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{15} = \frac{1}{v} - \frac{1}{12}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{15} + \frac{1}{12}$$

$$\Rightarrow \frac{1}{v} = \frac{4+5}{60}$$

$$\Rightarrow v = \frac{60}{9} = \frac{20}{3} = 6.67 \text{ cm} \quad \text{Ans}$$

$$m = \frac{h'}{h} = \frac{-v}{u} = \frac{-6.67}{-12} = \frac{6.67}{12} \quad \text{Ans}$$

$$3\overline{)20}\ (6.6$$
$$18$$
$$20$$
$$18$$

Q An object is placed in front of concave mirror of focal length 10cm. If image formed is double the height of object, find out position of object.

Sol. $f = -10$ cm

CASE-1 [Object is placed between C and F]

Real image $\Rightarrow h' = -2h$

$$\frac{h'}{h} = \frac{-v}{u} \qquad\qquad \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\Rightarrow \frac{-2h}{h} = \frac{-v}{u} \qquad\qquad \Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{2u} = \frac{3}{2u}$$

$$\Rightarrow v = 2u \qquad\qquad \Rightarrow \frac{1}{f} = \frac{3}{2u} \Rightarrow u = -\frac{30}{2} = -15 \text{ cm} \quad \text{Ans (i)}$$

$$\boxed{\Rightarrow -\frac{10}{3} = u \quad \Rightarrow u = -\frac{10}{3}\text{cm}}$$

CASE-2 [Object is placed between F and P]

Virtual image $\Rightarrow h' = 2h$

$$\frac{h'}{h} = \frac{-v}{u}$$

$$\Rightarrow \frac{2\cancel{h}}{\cancel{h}} = \frac{-v}{u}$$

$$\Rightarrow v = -2u$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{u} - \frac{1}{2u}$$

$$\Rightarrow \frac{1}{f} = \frac{2-1}{2u}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{2u}$$

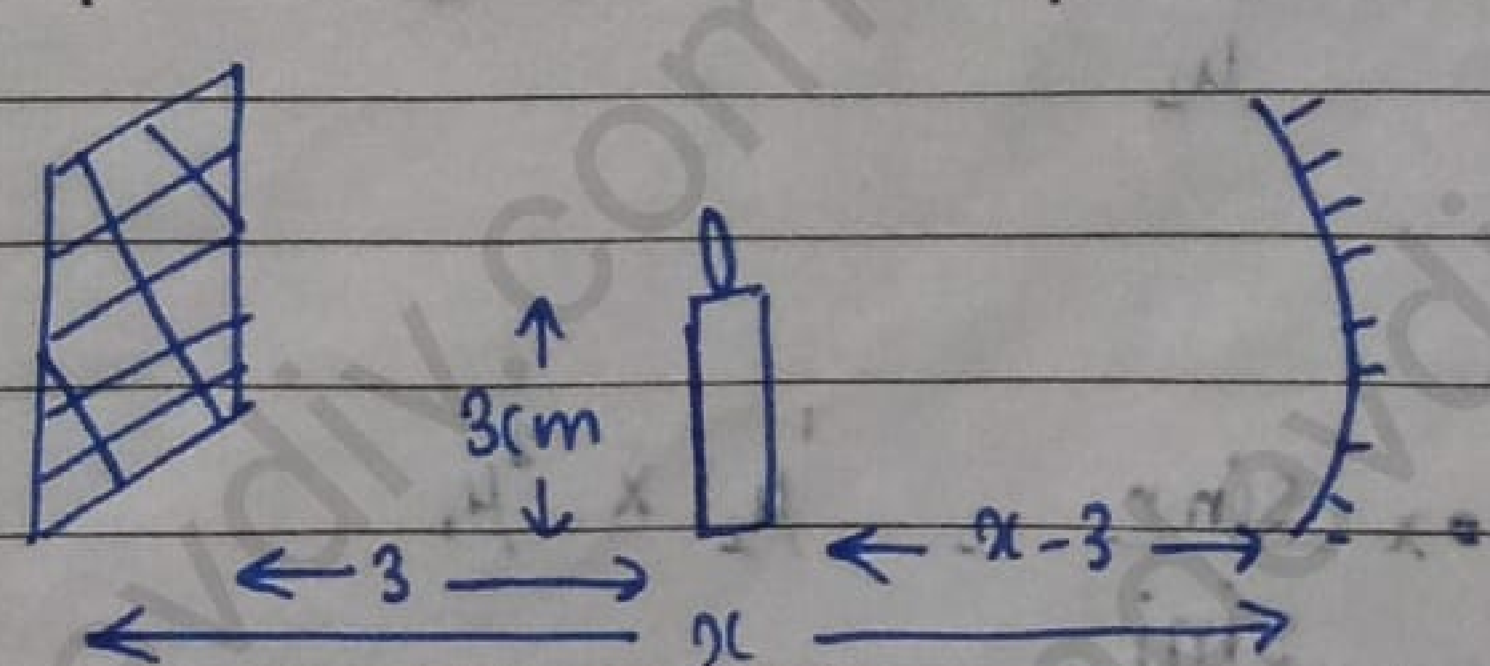
$$\Rightarrow f = 2u$$

$$\Rightarrow -10 = 2u$$

$$\Rightarrow u = -5\text{ cm}$$ Ans (ii)

Q A candle flame 3cm is placed at distance of 3m from a wall. How far from wall must a concave mirror is placed in order that it may form an image of flame 9cm high on the wall?

Sol.



$h = 3\text{cm}$

$h' = -9\text{ cm}$

$$\frac{h'}{h} = \frac{-v}{u}$$

$$\Rightarrow \frac{-9}{3} \times \frac{100}{100} = \frac{-(-x)}{-(x-3)}$$

$$\Rightarrow 3(x-3) = x$$

$$\Rightarrow 2x = 9$$

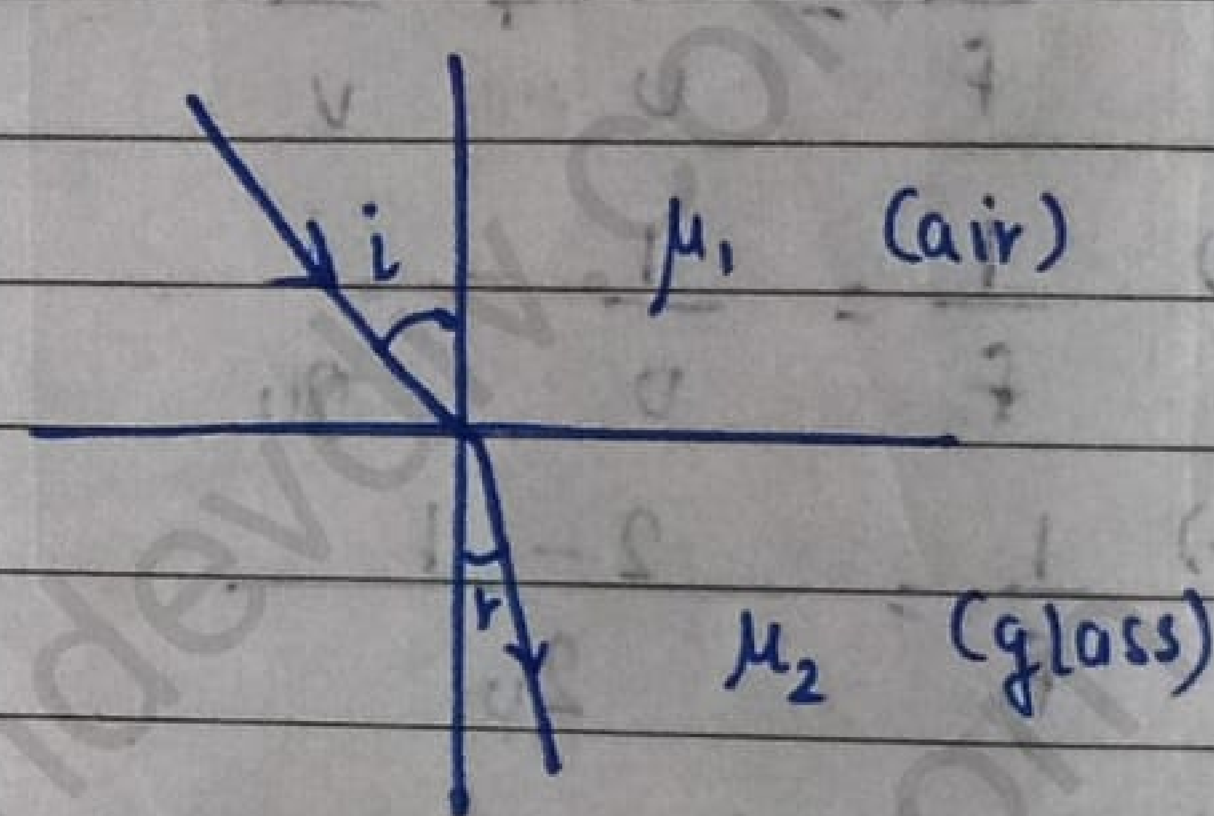
$$\Rightarrow x = \frac{9}{2}\,m = 4.5\,m$$

★ HOW TO APPLY SNELL'S LAW

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

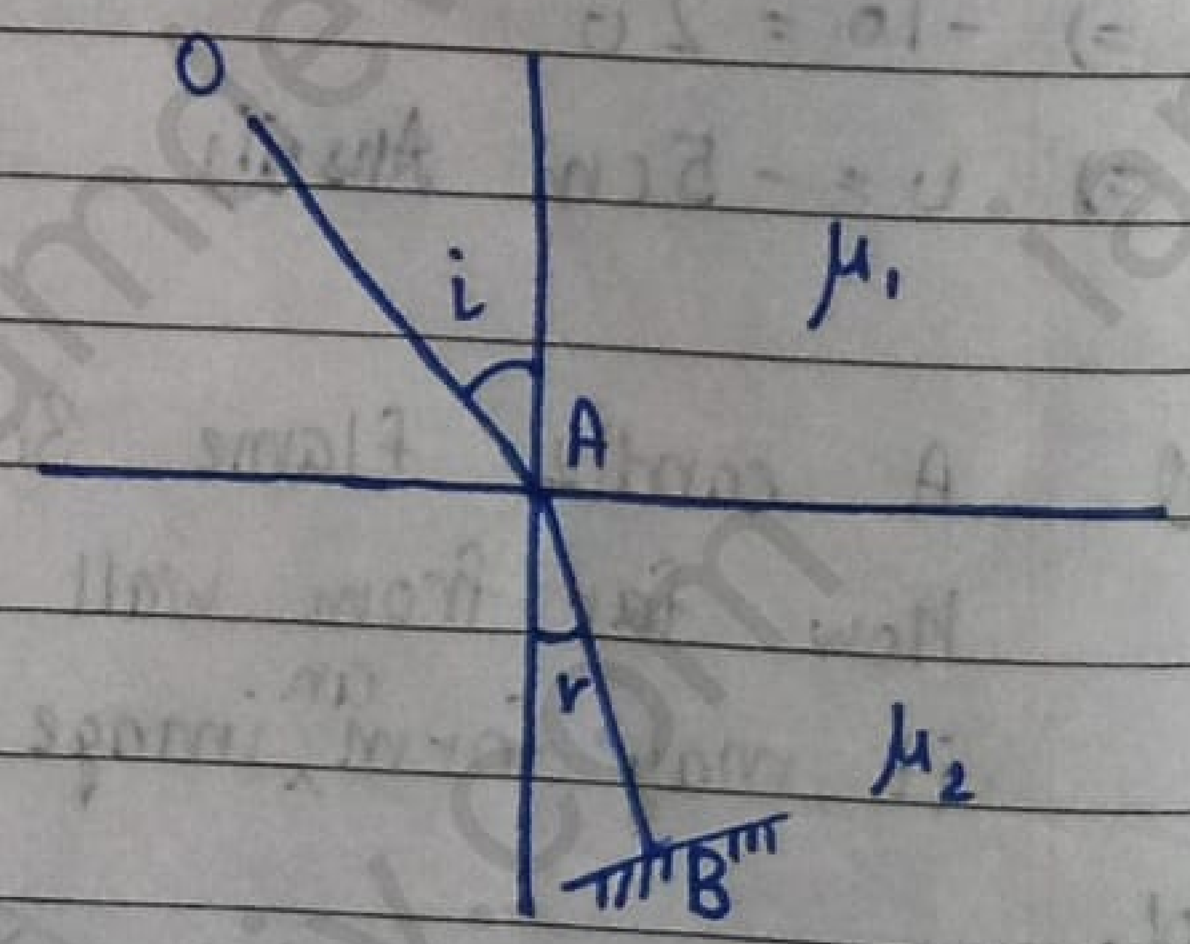
$$\frac{\sin i}{\sin r} = {}^{1}\mu_2$$



★ PRINCIPLE OF REVERSIBILITY

Applying Snell's law on path OAB

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = {}^{1}\mu_2 \quad \text{—— (1)}$$



$\mu_2 > \mu_1$

Applying snell's law on path BAO

$$\frac{\sin r}{\sin i} = \frac{\mu_1}{\mu_2} = {}^{2}\mu_1 \quad \text{—— (2)}$$

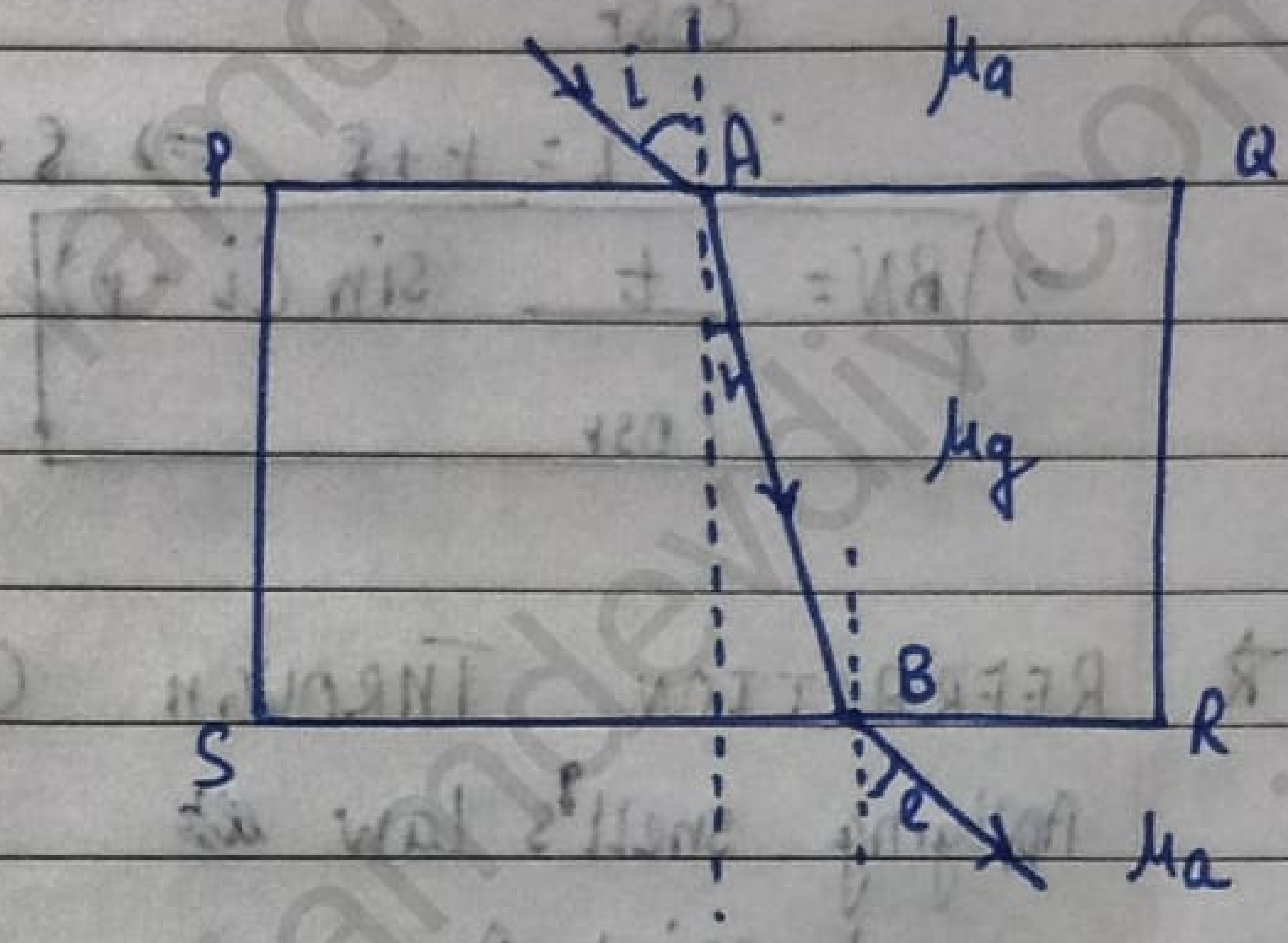
(1) × (2)

$$\frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i} = {}^{1}\mu_2 \times {}^{2}\mu_1$$

$$\Rightarrow 1 = {}^{1}\mu_2 \times {}^{2}\mu_1$$

$$\Rightarrow {}^{1}\mu_2 = \frac{1}{{}^{2}\mu_1} \quad \text{(OR)} \quad {}^{2}\mu_1 = \frac{1}{{}^{1}\mu_2}$$

★ REFRACTION THROUGH GLASS SLAB



Apply Snell's law at point A

$$\frac{\sin i}{\sin r} = \frac{\mu_g}{\mu_a} \quad \text{—— ①}$$

Apply Snell's law at point B

$$\frac{\sin r}{\sin e} = \frac{\mu_a}{\mu_g} \quad \text{—— ②}$$

① × ②

$$\frac{\sin i}{\cancel{\sin r}} \times \frac{\cancel{\sin r}}{\sin e} = \frac{\mu_g}{\mu_a} \times \frac{\mu_a}{\mu_g}$$

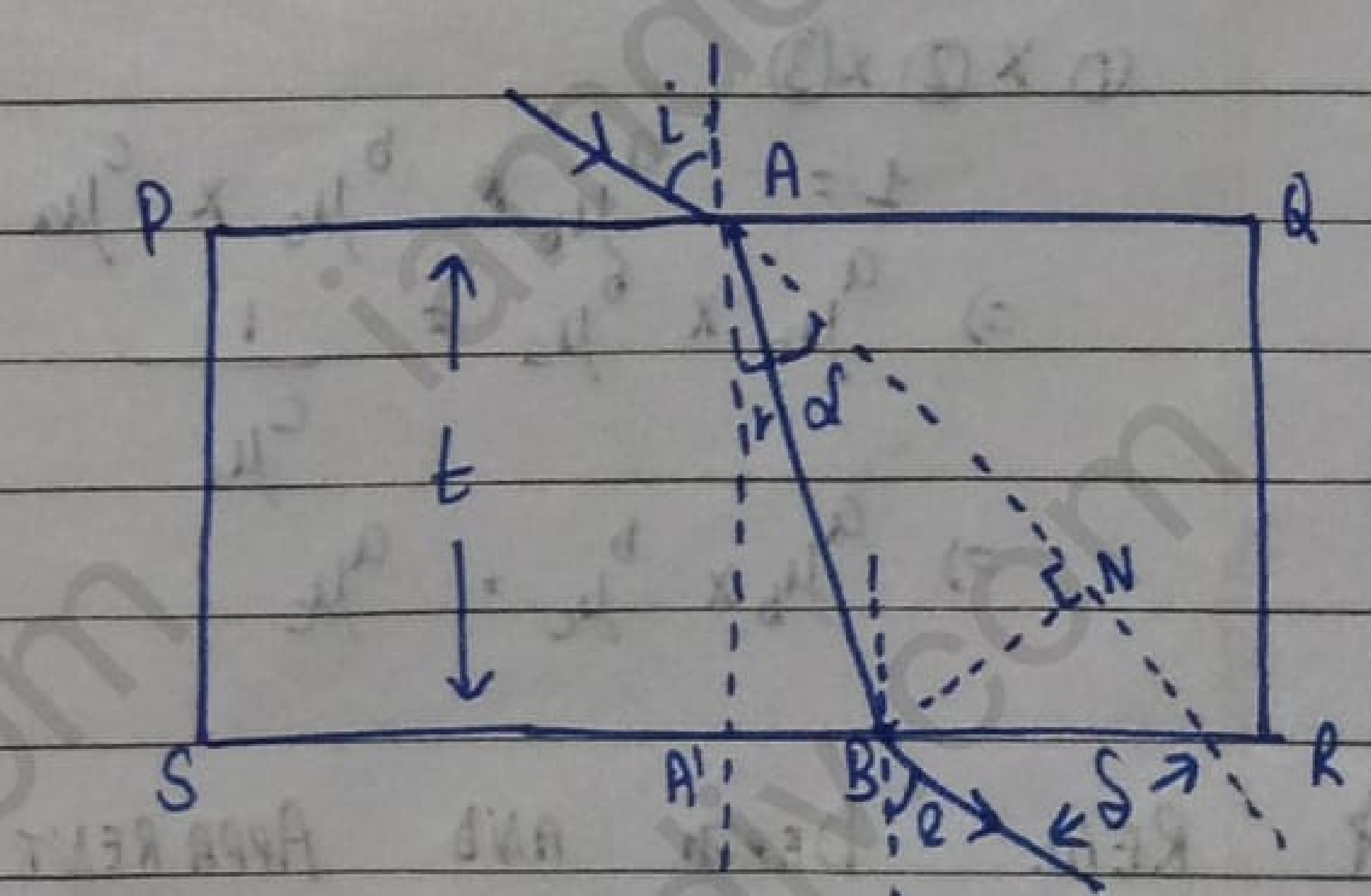
$\Rightarrow \dfrac{\sin i}{\sin e} = 1$

$\Rightarrow \sin i = \sin e$

$\Rightarrow \sin^{-1} \sin i = \sin^{-1} \sin e$

$\Rightarrow i = e$

★ EXPRESSION FOR LATERAL DISPLACEMENT



In ΔABN,

$\sin \delta = \dfrac{BN}{AB}$

$BN = AB \sin\delta$ —— ①

In $\Delta AA'B$

$\cos r = \dfrac{AA'}{AB} = \dfrac{t}{AB}$

$AB = \dfrac{t}{\cos r}$

Put this value in ①

$$BN = \frac{t}{\cos r} \sin \delta$$

$\because i = r + \delta \Rightarrow \delta = i - r$

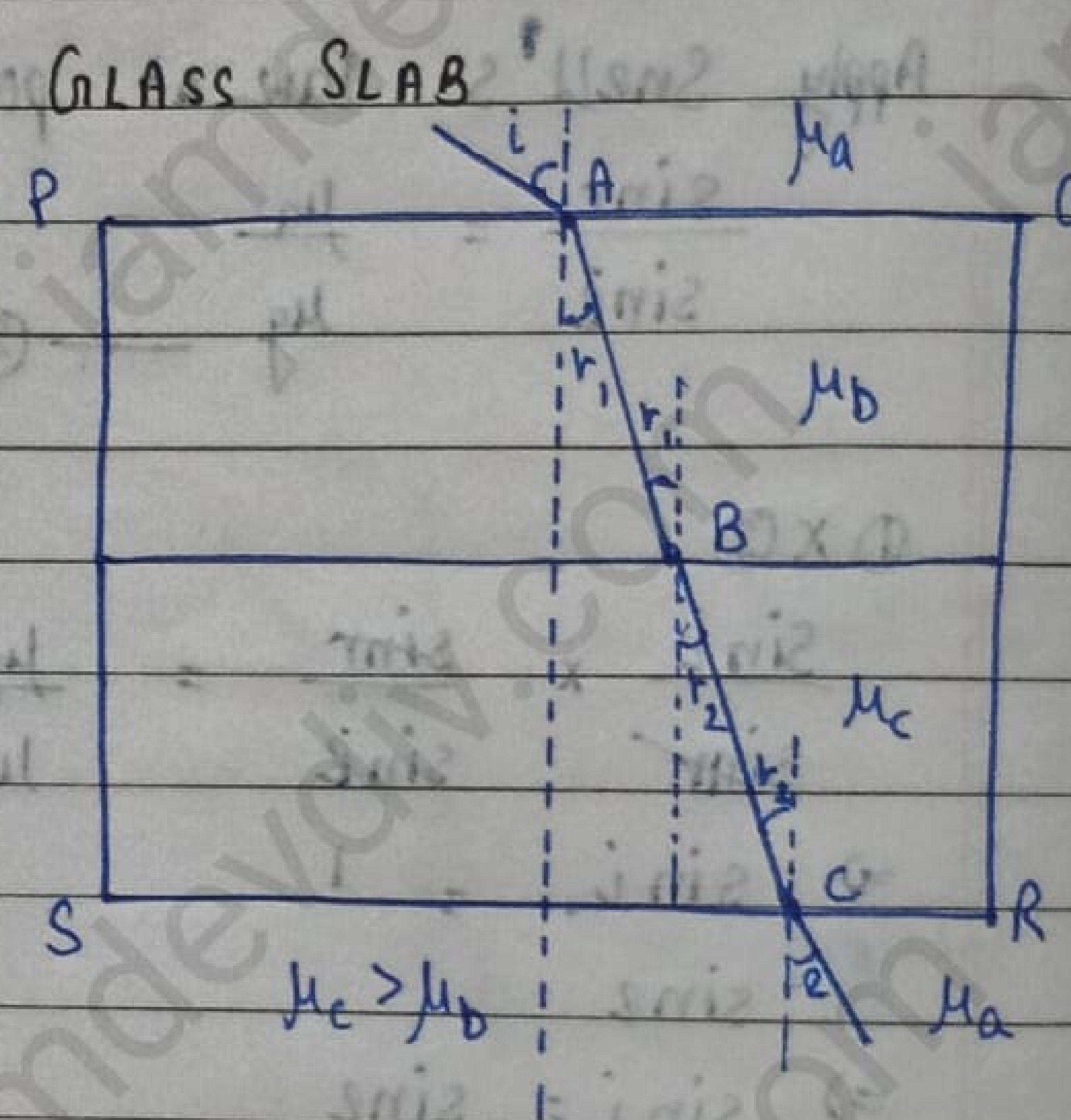
$$\Rightarrow \boxed{BN = \frac{t}{\cos r} \sin (i - r)}$$

★ REFRACTION THROUGH COMPOUND GLASS SLAB

Applying snell's law

at point A

$$\frac{\sin i}{\sin r_1} = \frac{\mu_b}{\mu_a} = {}^a\mu_b \quad \text{— ①}$$



at point B

$$\frac{\sin r_1}{\sin r_2} = \frac{\mu_c}{\mu_b} = {}^b\mu_c \quad \text{— ②}$$

at point C

$$\frac{\sin r_2}{\sin i} = \frac{\mu_a}{\mu_c} = {}^c\mu_a \quad \text{— ③}$$

① × ② × ③

$$1 = {}^a\mu_b \times {}^b\mu_c \times {}^c\mu_a$$

$$\Rightarrow {}^a\mu_b \times {}^b\mu_c = \frac{1}{{}^c\mu_a}$$

$$\Rightarrow {}^a\mu_b \times {}^b\mu_c = {}^a\mu_c$$

★ REAL DEPTH AND APPARENT DEPTH

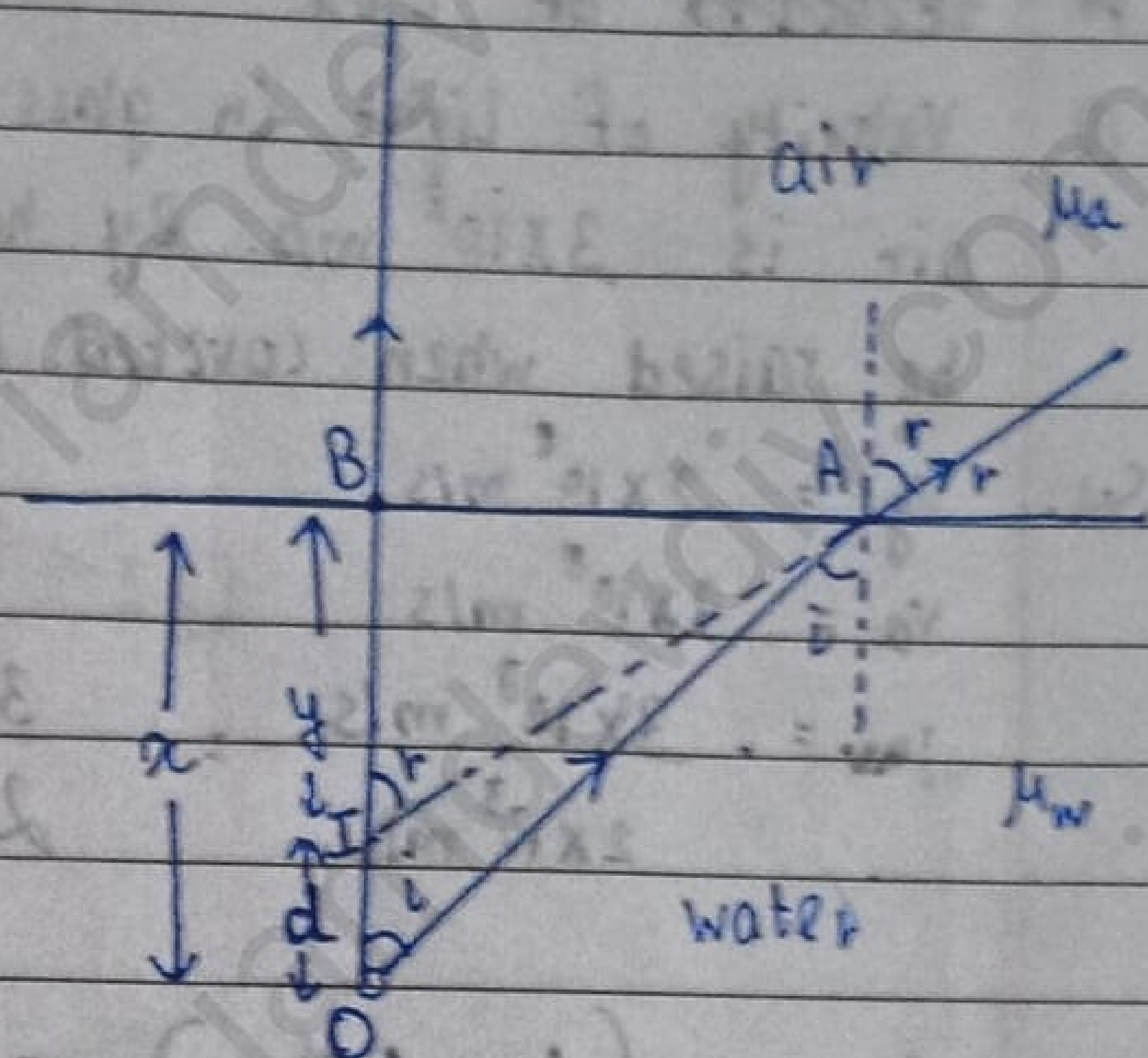
$$\frac{\sin i}{\sin r} = \frac{\mu_a}{\mu_w} = {}^w\mu_a \quad \text{— ①}$$

In ΔABO

$$\sin i = \frac{AB}{OA}$$

In ΔABI,

$$\sin r = \frac{AB}{AI}$$



Put these values in (1),

$$\frac{\frac{AB}{OA}}{\frac{AB}{AI}} = {}^{w}\mu_a$$

$$\Rightarrow \frac{AI}{OA} = {}^{w}\mu_a$$

If angles are very small then A lies very close to B,

$$\frac{BI}{OB} = {}^{w}\mu_a$$

$$\Rightarrow \frac{y}{x} = {}^{w}\mu_a$$

$$\Rightarrow \frac{x}{y} = {}^{a}\mu_w$$

$$\Rightarrow \mu_w = \frac{x}{y} \quad \begin{matrix} \to \text{Real image} \\ \to \text{Apparent image} \end{matrix}$$

Normal shift $\Rightarrow$ $d = x - y$

$$\Rightarrow d = x - \frac{x}{\mu_w}$$

$$\Rightarrow d = \cancel{E}\ x\left(1 - \frac{1}{\mu_w}\right).$$

Q A glass having refractive index 1.5 and having thickness 4cm is placed on an ink dot on a paper. Find out rise in the image of ink dot.

Sol. $d = 4\left(1 - \frac{1}{1.5}\right)$

$$\Rightarrow d = 4\left(\frac{1.5 - 1}{1.5}\right) \Rightarrow 4 \times \frac{\cancel{0.5}}{\cancel{1.5}_3} \Rightarrow \frac{4}{3}\text{ cm}$$

★ ~~Velocity of Light~~

Q Velocity of Light in glass is 2×10^8 m/s and velocity of Light in air is 3×10^8 m/s. By how much would an ink dot appear to be raised when covered by glass plate 6 cm thick.

Sol. $v_g = 2 \times 10^8$ m/s

$v_a = 3 \times 10^8$ m/s

$$\mu = \frac{3 \times 10^8 \text{ m/s}}{2 \times 10^8 \text{ m/s}} = \frac{3}{2} = 1.5$$

$$d = x\left(1 - \frac{1}{\mu}\right) = 6\left(1 - \frac{1}{1.5}\right) = 6\left(\frac{1.5 - 1}{1.5}\right) = \frac{6 \times 0.5}{1.5}$$

$= 2$ cm Ans

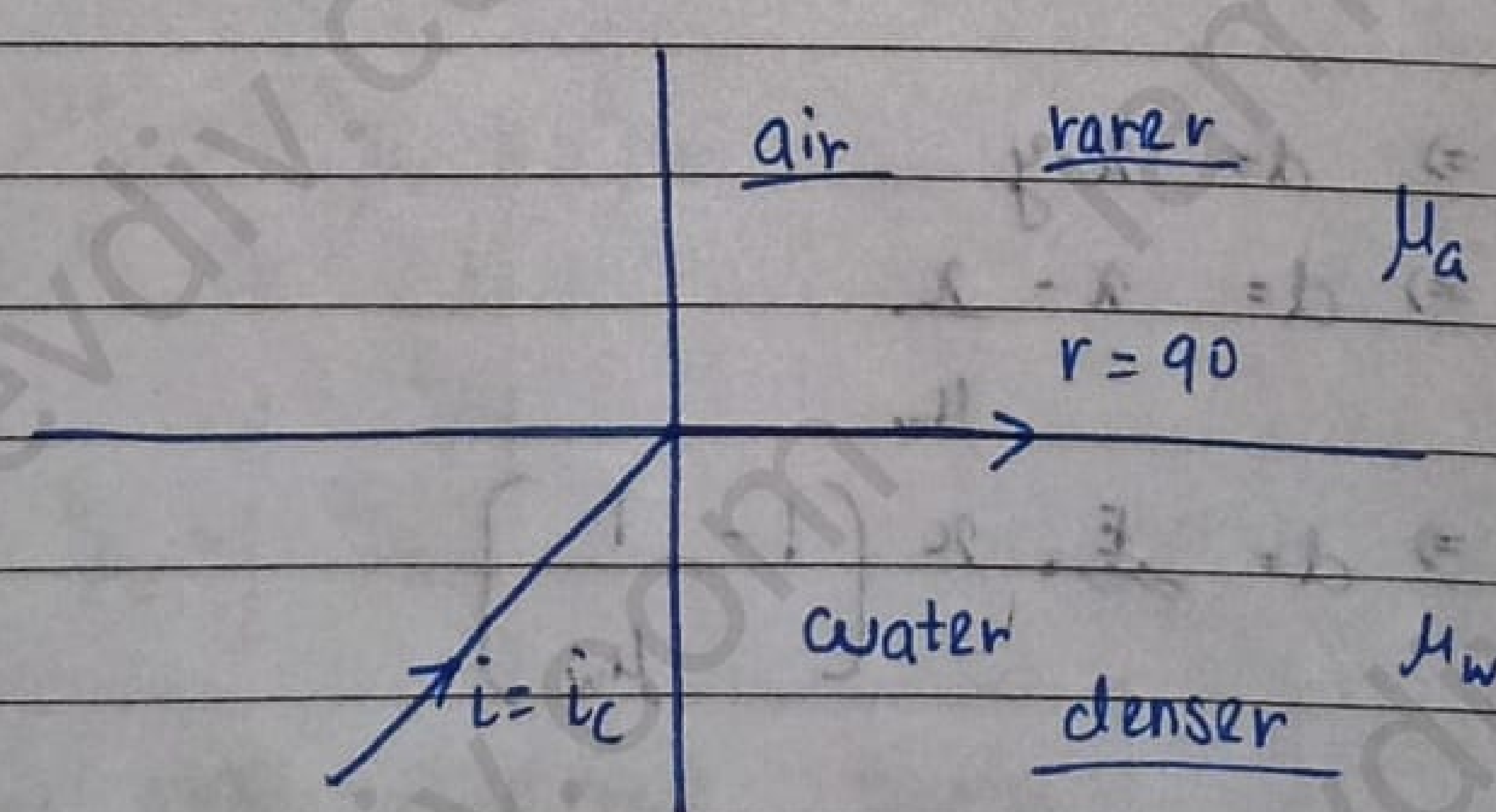
~~$d = x - y$~~

~~$\Rightarrow 2 = 6 - y$~~

~~$\Rightarrow y = 4$ cm Ans~~

★ CRITICAL ANGLE

It is the angle of incidence in denser medium for which angle of refraction in rarer medium is $90°$.



By snell's law,

$$\frac{\sin i}{\sin r} = \frac{\mu_a}{\mu_w} = {}^w\mu_a$$

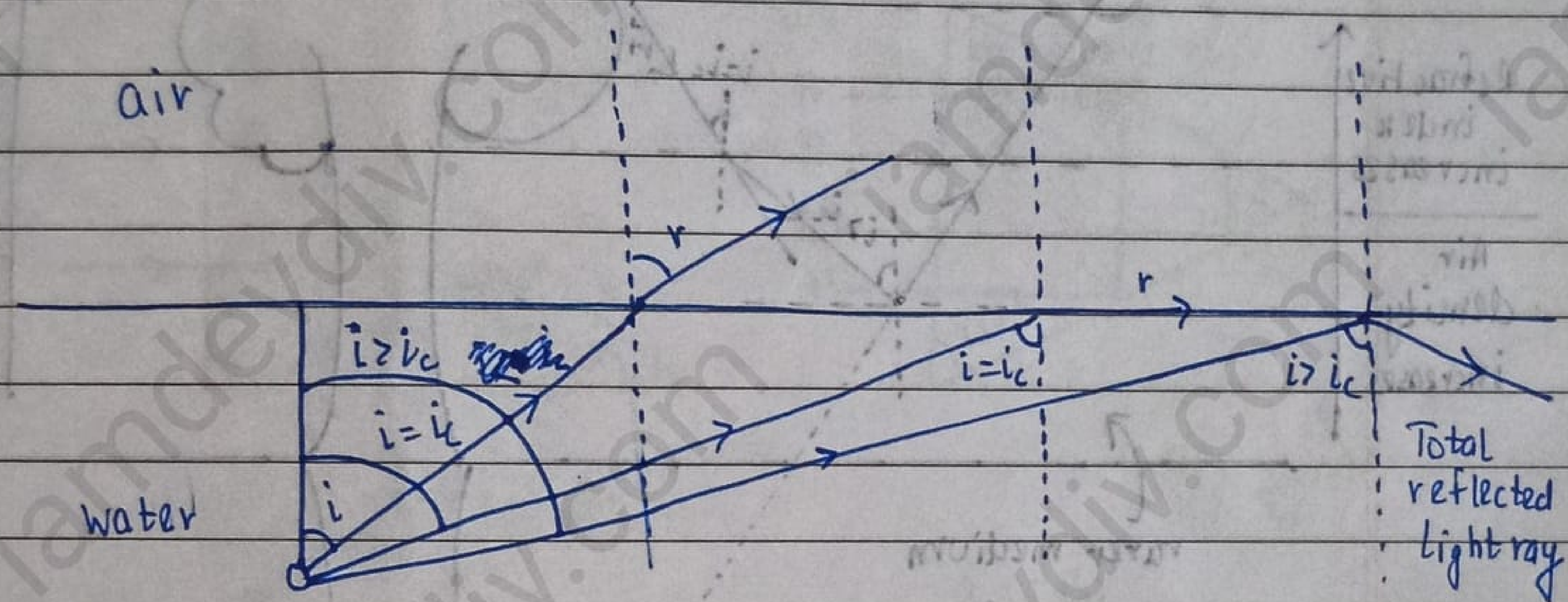
but $i = i_c$, $r = 90°$

$$\Rightarrow \frac{\sin i_c}{\sin 90^\circ} = {}^{w}\mu_a$$

$$\Rightarrow \sin i_c = {}^{w}\mu_a$$

$$\Rightarrow \boxed{i_c = \sin^{-1}({}^{w}\mu_a)}$$

★ TOTAL INTERNAL REFLECTION [TIR]



When a ray of light strikes the interface at an angle greater than critical angle, it comes back in the same medium. This phenomena is called TIR.

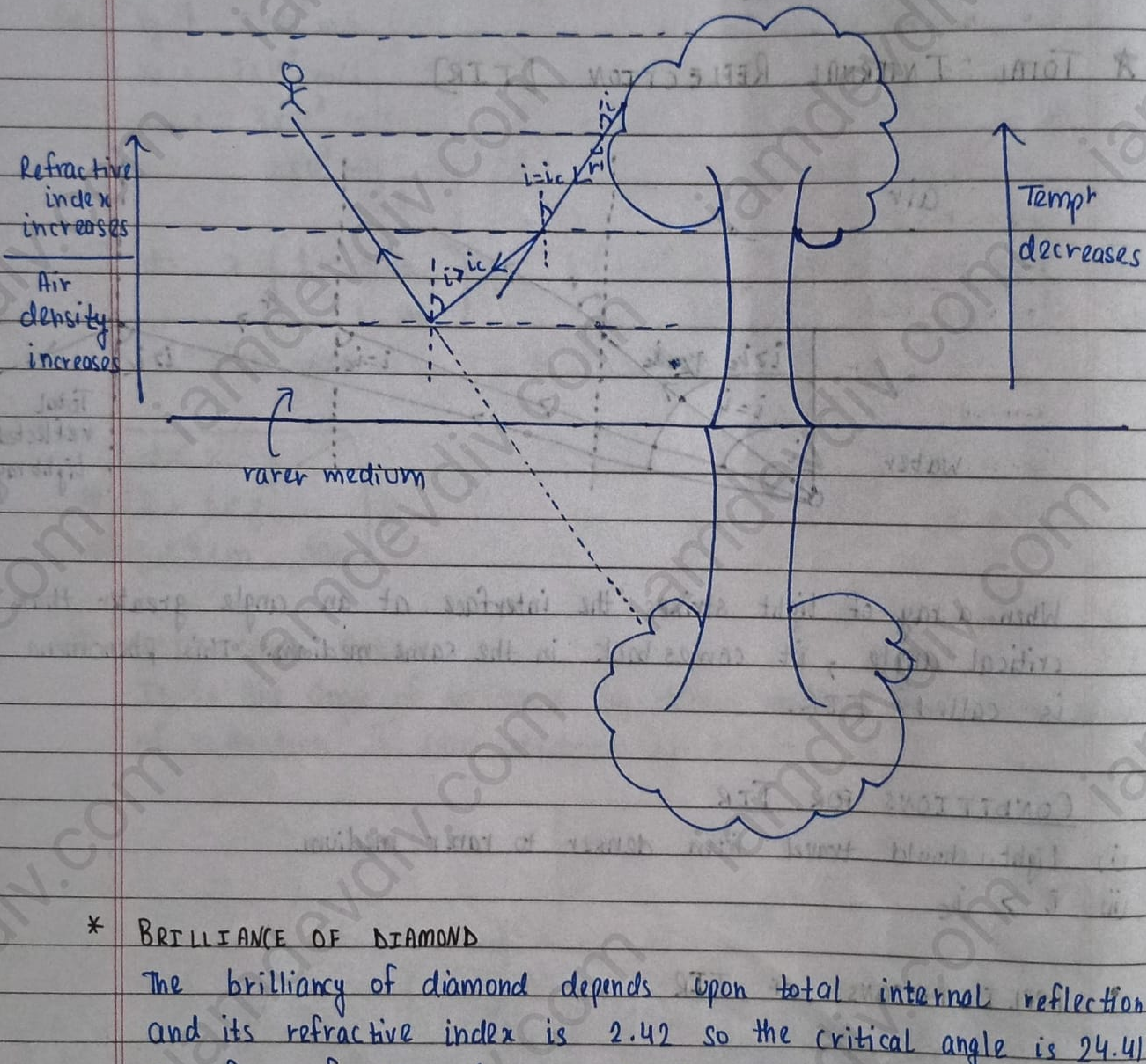
CONDITIONS FOR TIR

(i) Light should travel from denser to rarer medium.
(ii) $i > i_c$

• APPLICATIONS OF TIR

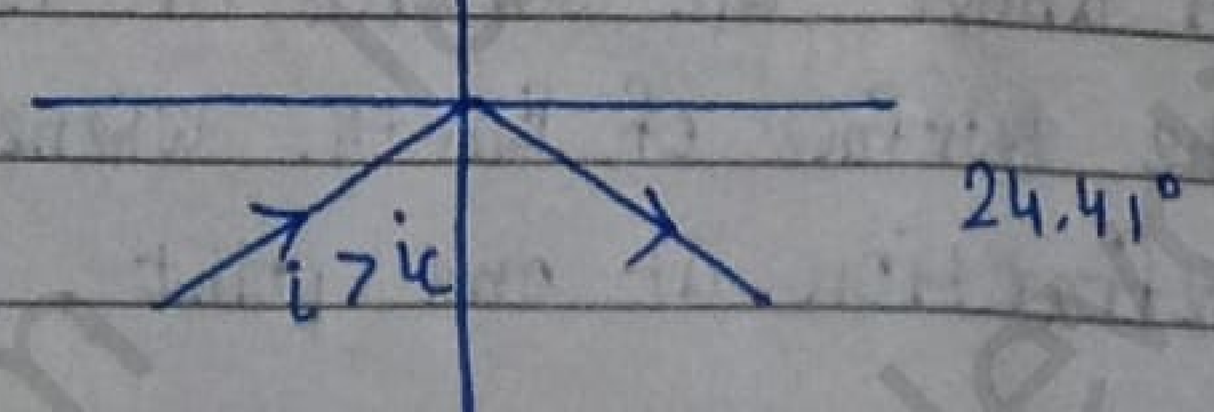
* MIRAGE

Mirage is an optical illusion of deserts on hot summer days. Due to very high temperature, the air near the surface of Earth becomes rarer and above the surface of Earth will be denser as compared to near the surface of Earth. Whenever a ray coming from tree undergo refraction at one point, the ray makes an angle greater than critical angle. It undergoes in TIR and when these light rays goes ~~to~~ into the person's eye, an inverted image of tree is seen by person and ~~appear~~ tree appears as water but actually there is no water.



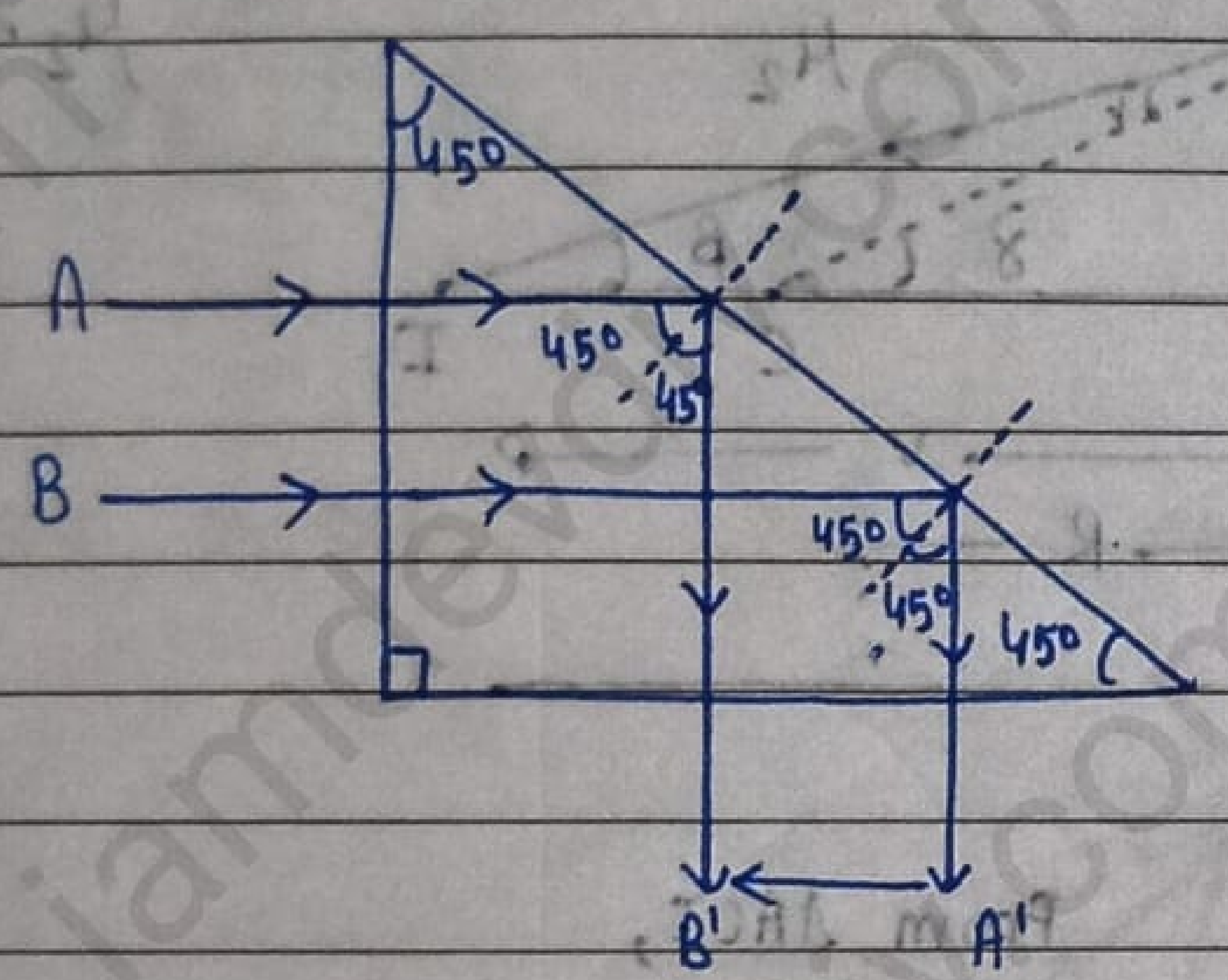
* BRILLIANCE OF DIAMOND

The brilliancy of diamond depends upon total internal reflection and its refractive index is 2.42 so the critical angle is 24.41°. The faces of diamond are cut in such a way that the angle of incidence ~~of~~ is greater than critical angle and light comes back in the same medium and suffer multiple TIRs. In this way, diamond shines.

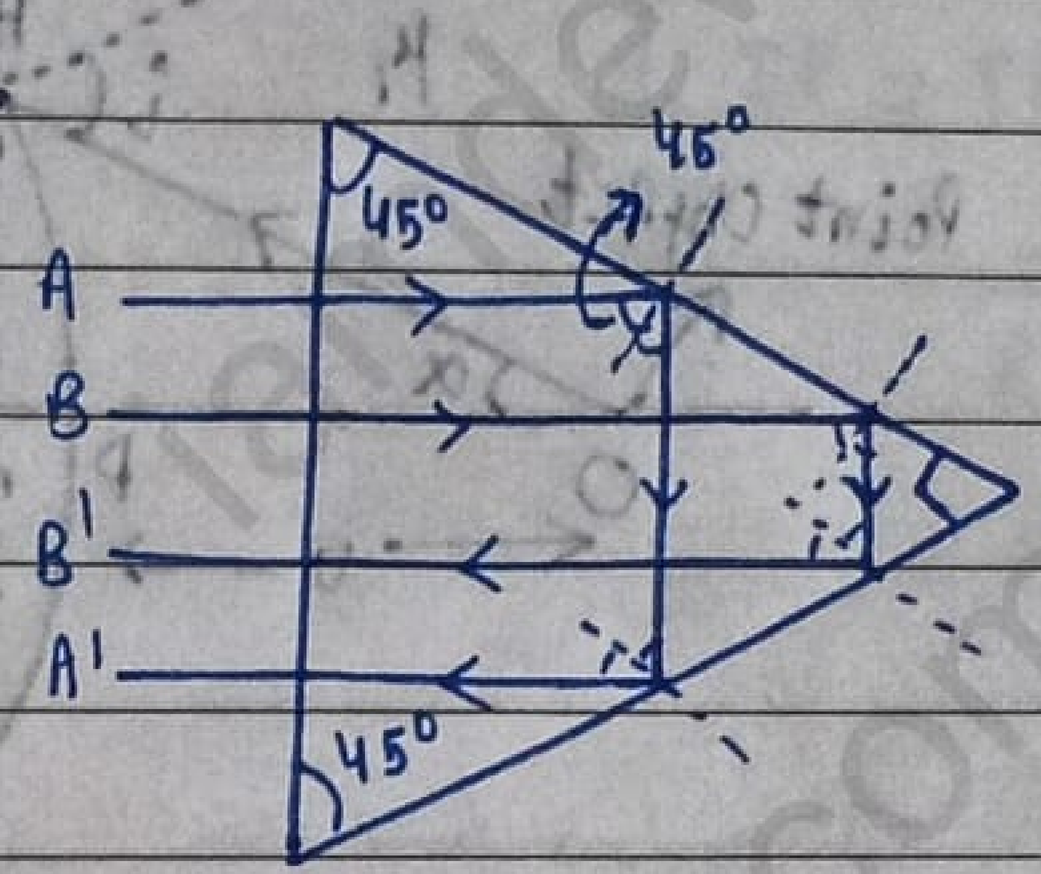


* TOTALLY REFLECTING GLASS PRISM

For glass and air interface, the critical angle is 42° and the faces of prism are cut in such a way that when the light strikes the surface of prism, it makes an angle of 45° which is greater than critical angle and therefore, light suffer TIR and comes out from the prism and different images are formed.

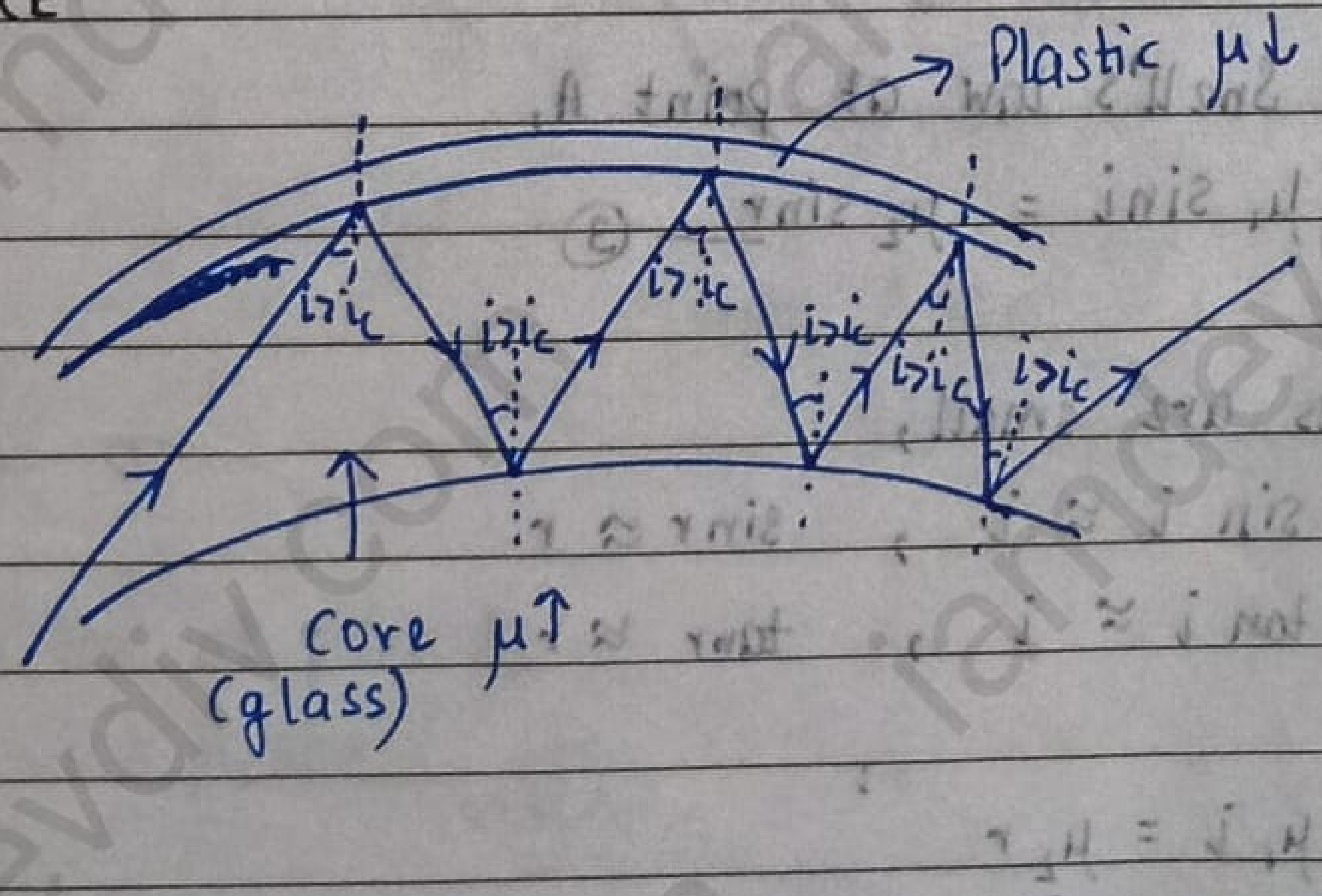


(a)



(b)

* OPTICAL FIBRE

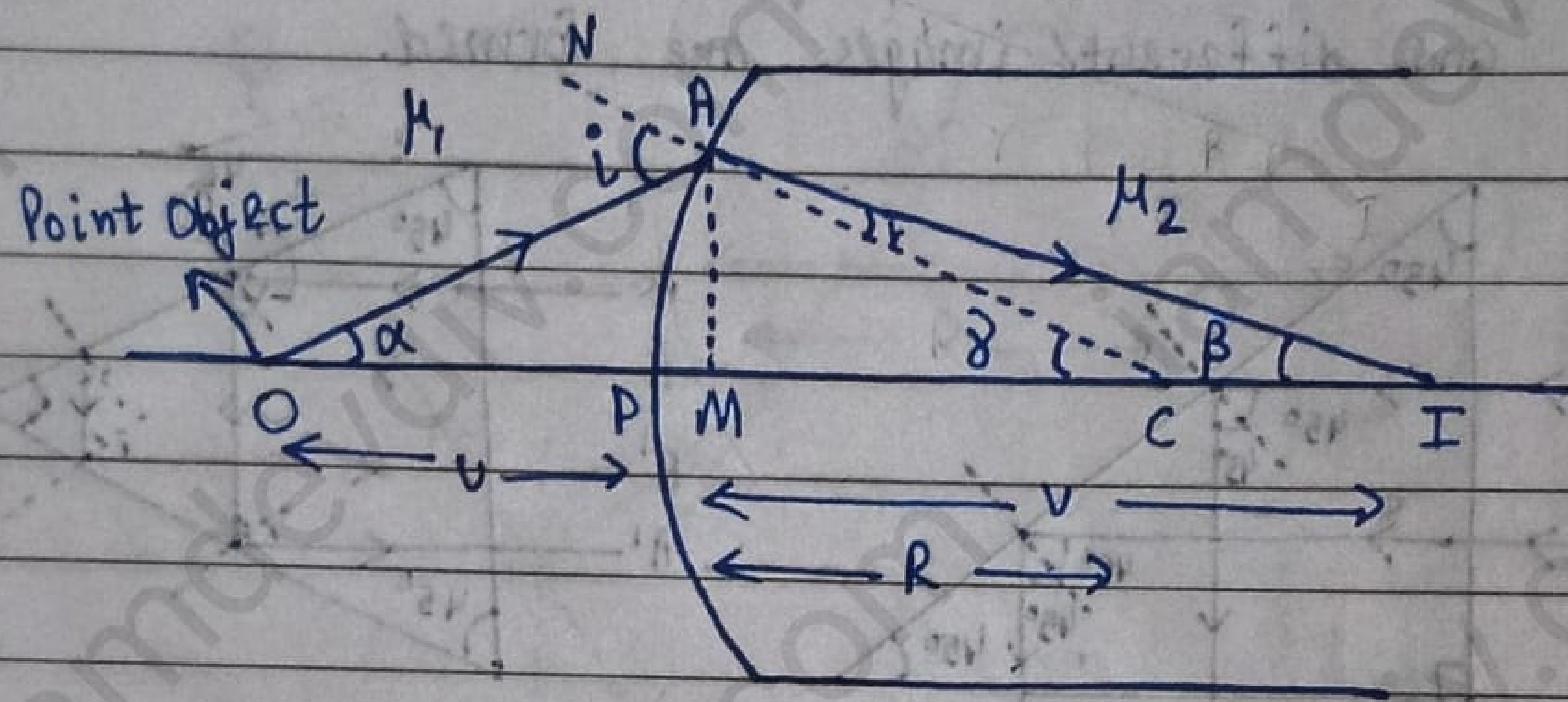


It is a device which is used to transmit the light from one end to another. It consists of two layers, one is called inner layer called core and another is outer layer called cladding. The core has high refractive index while cladding has low refrac~~tive~~ ~~index~~ index. When light coming from one end is incident on core at an angle greater than critical angle, it suffers multiple TIR and comes out from other end.

☆ REFRACTION THROUGH SPHERICAL REFRACTING SURFACES

- CONVEX SPHERICAL REFRACTING SURFACE

* RARER TO DENSER [REAL IMAGE]



$\mu_2 > \mu_1$

From ΔAOC,

$$i = \alpha + \gamma \quad \text{—①}$$

From ΔACI,

$$\gamma = r + \beta$$
$$\Rightarrow r = \gamma - \beta \quad \text{—②}$$

Applying Snell's law at point A,

$$\mu_1 \sin i = \mu_2 \sin r \quad \text{—③}$$

If angles are small,

$$\sin i \approx i \quad , \quad \sin r \approx r$$
$$\tan i \approx i \quad , \quad \tan r \approx r$$

$$③ \Rightarrow \mu_1 i = \mu_2 r$$
$$\Rightarrow \mu_1 (\alpha + \gamma) = \mu_2 (\gamma - \beta) \qquad \text{[From ① and ②]}$$
$$\boxed{\cancel{\Rightarrow \alpha \mu_1 + \gamma \mu_1 = \gamma \mu_2 - \mu_2 \beta}}$$
$$\Rightarrow \mu_1 (\tan \alpha + \tan \gamma) = \mu_2 (\tan \gamma - \tan \beta) \quad \text{—④}$$

From $\Delta AOM \Rightarrow \tan \alpha = \alpha = \dfrac{AM}{-u}$

From $\Delta AIM \Rightarrow \tan \beta = \beta = \dfrac{AM}{v}$

From $\Delta ACM \Rightarrow \tan\gamma = \gamma = \frac{AM}{R}$

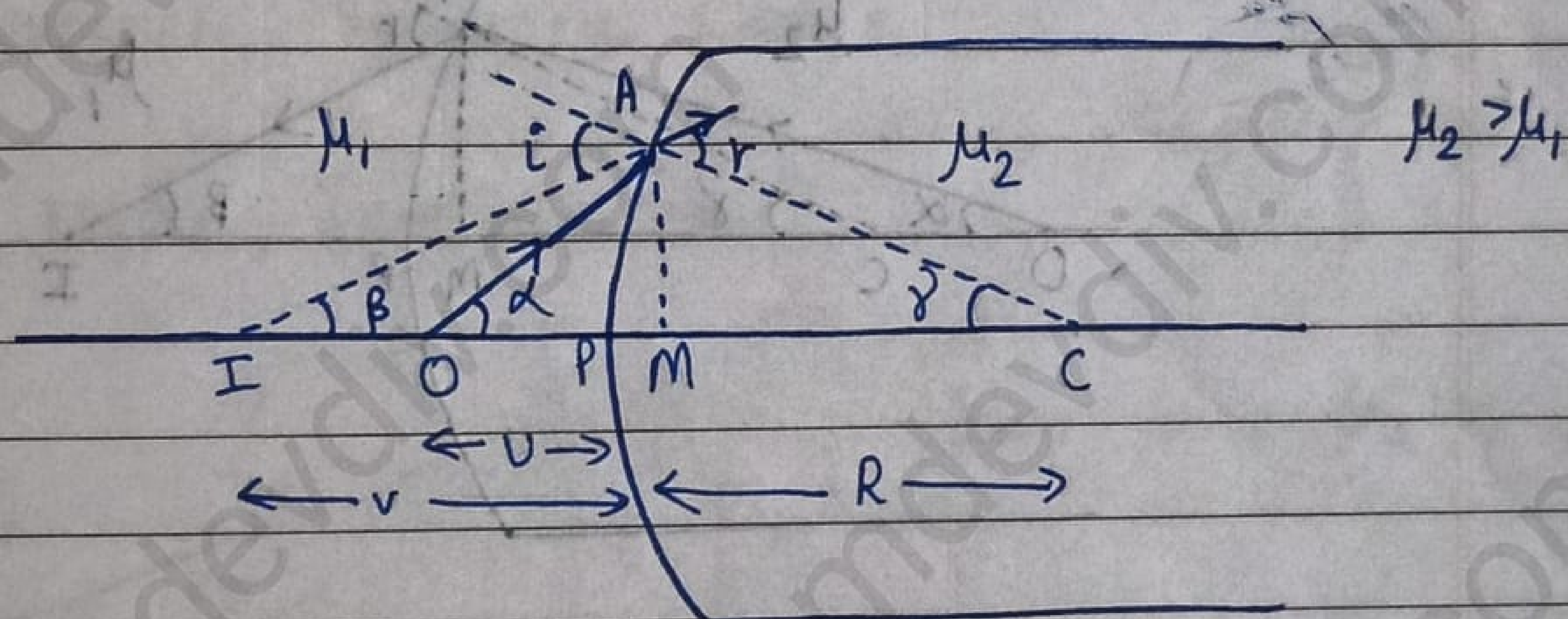
④ $\Rightarrow \mu_1\left(\frac{AM}{-u} + \frac{AM}{R}\right) = \mu_2\left(\frac{AM}{R} - \frac{AM}{v}\right)$

[$\because PM \Rightarrow$ small distance
$OP \simeq OM = -u$
$PC \simeq CM = V$
$IP \simeq IM = r$]

$\Rightarrow \frac{\mu_1}{R} - \frac{\mu_1}{u} = \frac{\mu_2}{R} - \frac{\mu_2}{v}$

$\Rightarrow \boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$ (OR) $-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$

* RARER TO DENSER [VIRTUAL IMAGE]



From ΔAOC,
$i = \alpha + \gamma$ — ①

From ΔAIC,
$r = \beta + \gamma$ — ②

Applying Snell's law at point A,
$\mu_1 \sin i = \mu_2 \sin r$ — ③

If angles are small,
$\sin i \simeq i$, $\sin r \simeq r$
$\tan i \simeq i$, $\tan r \simeq r$

③ $\Rightarrow \mu_1 i = \mu_2 r$
$\Rightarrow \mu_1(\alpha + \gamma) = \mu_2(\beta + \gamma)$ [from ① and ②]
$\Rightarrow \mu_1(\tan\alpha + \tan\gamma) = \mu_2(\tan\beta + \tan\gamma)$

$$\Rightarrow \mu_1\left(\frac{AM}{OM} + \frac{AM}{MC}\right) = \mu_2\left(\frac{AM}{IM} + \frac{AM}{MC}\right)$$

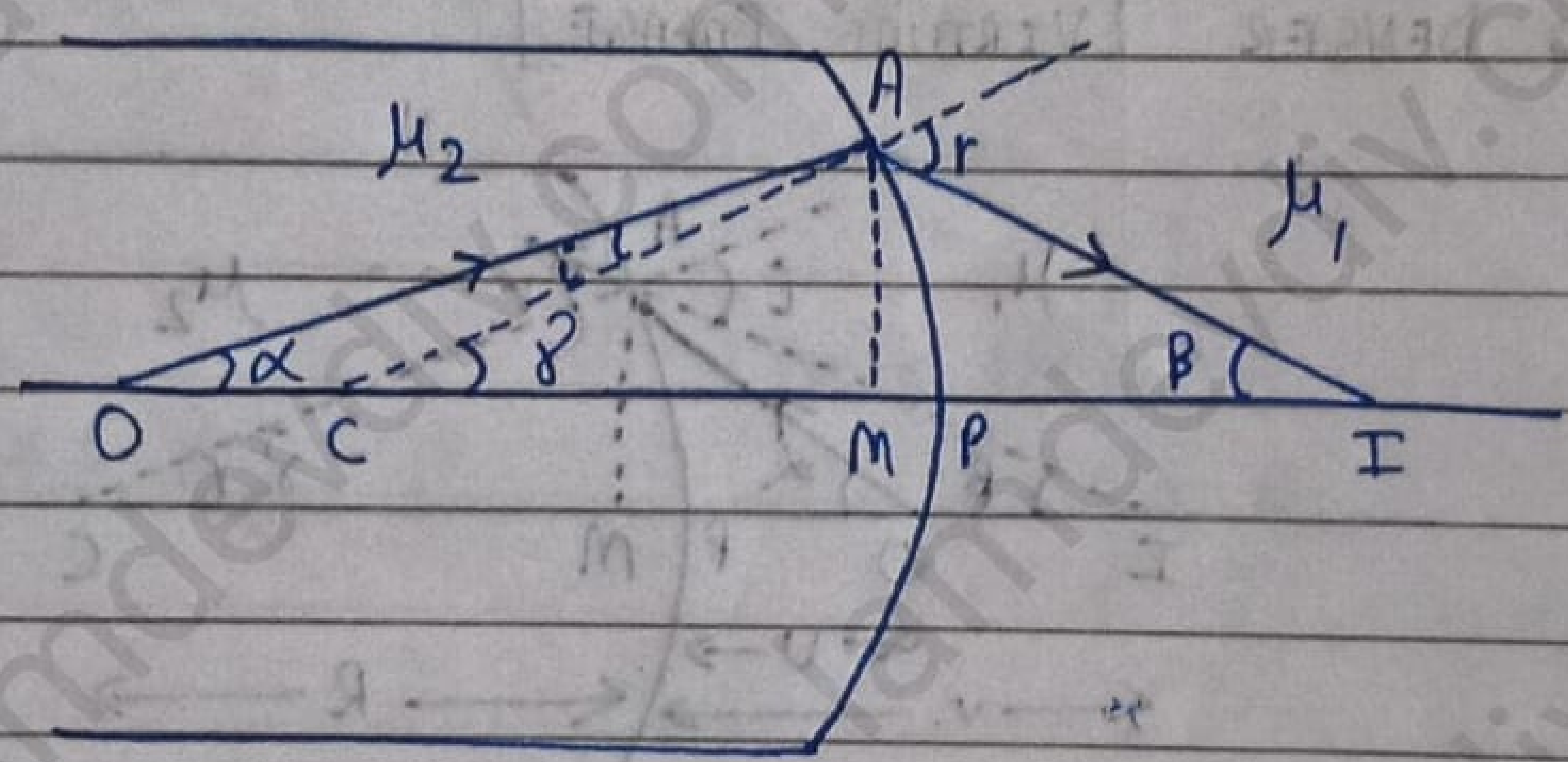
$$\Rightarrow \mu_1\left(\frac{1}{-u} + \frac{1}{R}\right) = \mu_2\left(\frac{1}{-v} + \frac{1}{R}\right)$$

$$\Rightarrow \frac{\mu_1}{-u} + \frac{\mu_1}{R} = \frac{\mu_2}{-v} + \frac{\mu_2}{R}$$

$$\Rightarrow \boxed{\frac{\mu_1}{u} - \frac{\mu_2}{v} = \frac{\mu_1 - \mu_2}{R}}$$

[$\because$ OM ⇒ small distance
$OP \simeq OM = -u$
$IP \simeq IM = -v$
$PC \simeq CM = R$]

* DENSER TO RARER [REAL IMAGE]



$\mu_2 > \mu_1$

From ΔAOC,
$$i + \alpha = \gamma$$
$$\Rightarrow i = \gamma - \alpha \quad \text{--- (1)}$$

From ΔACI,
$$r = \gamma + \beta \quad \text{--- (2)}$$

Applying Snell's law at point A,
$$\mu_2 \sin i = \mu_1 \sin r \quad \text{--- (3)}$$

If angles are small,
$$\sin i \simeq i \quad , \quad \sin r \simeq r$$
$$\tan i \simeq i \quad , \quad \tan r \simeq r$$

(3) $\Rightarrow \mu_2 i = \mu_1 r$
$$\Rightarrow \mu_2(\gamma - \alpha) = \mu_1(\gamma + \beta)$$
$$\Rightarrow \mu_2(\tan\gamma - \tan\alpha) = \mu_1(\tan\gamma + \tan\beta)$$

$$\Rightarrow \mu_2\left(\frac{AM}{CM} - \frac{AM}{OM}\right) = \mu_1\left(\frac{AM}{CM} - \frac{AM}{MI}\right)$$

$$\Rightarrow \mu_2\left(\frac{1}{-R} + \frac{1}{u}\right) = \mu_1\left(\frac{1}{-R} + \frac{1}{v}\right)$$

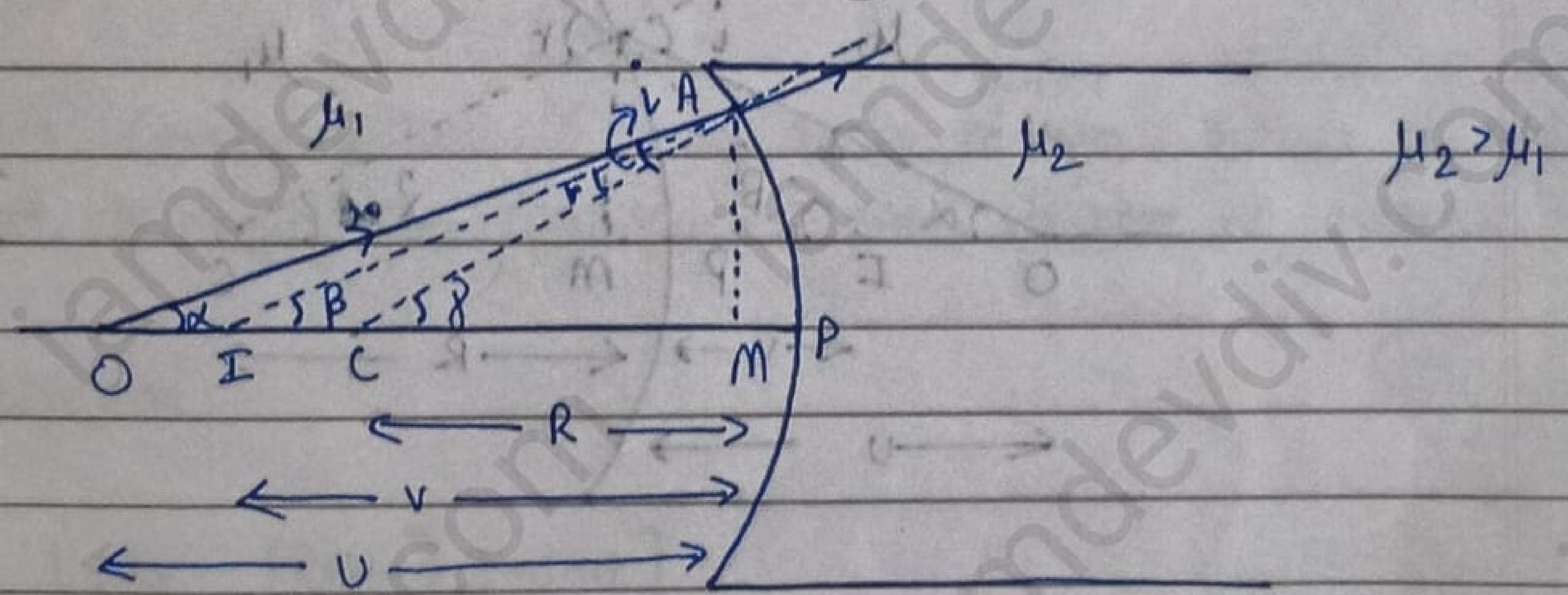
$$\Rightarrow \frac{\mu_2}{-R} + \frac{\mu_2}{u} = \frac{\mu_1}{-R} + \frac{\mu_1}{v}$$

$$\Rightarrow \frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

$$\Rightarrow \boxed{-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}}$$

- CONCAVE SPHERICAL REFRACTING SURFACE

* RARER TO DENSER [VIRTUAL IMAGE]



From ΔAOC,

$$i + \alpha = \gamma$$

$$\Rightarrow i = \gamma - \alpha \quad \text{---} (1)$$

From ΔAIC,

$$r + \beta = \gamma$$

$$\Rightarrow r = \gamma - \beta \quad \text{---} (2)$$

Apply Snell's law at point A,

$$\mu_1 \sin i = \mu_2 \sin r \quad \text{---} (3)$$

If angles are small,

$$\sin i \simeq i \quad , \quad \sin r \simeq r$$

$$\tan i \simeq i \quad , \quad \tan r \simeq r$$

③ $\Rightarrow \mu_1 i = \mu_2 r$

$\Rightarrow \mu_1(\gamma - \alpha) = \mu_2(\gamma - \beta)$ [From ① and ②]

$\Rightarrow \mu_1(\tan\gamma - \tan\alpha) = \mu_2(\tan\gamma - \tan\beta)$

$\Rightarrow \mu_1\left(\frac{AM}{CM} - \frac{AM}{OM}\right) = \mu_2\left(\frac{AM}{CM} - \frac{AM}{IM}\right)$

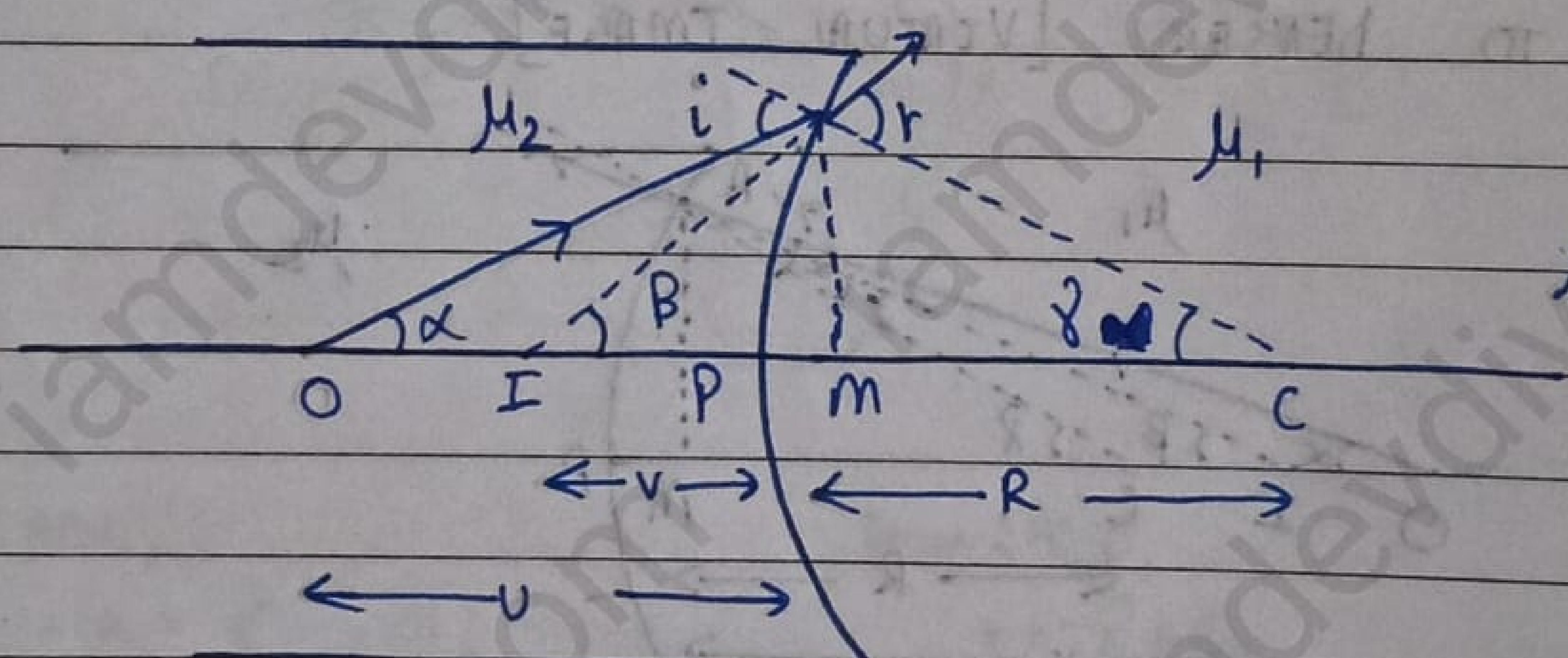
$\Rightarrow \mu_1\left(\frac{1}{-R} + \frac{1}{u}\right) = \mu_2\left(\frac{1}{-R} + \frac{1}{v}\right)$

[$\because$ PM $\Rightarrow$ small distance
OP $\approx$ OM = $-u$
CP $\approx$ CM = $-R$
IP $\approx$ IM = $-v$]

$\Rightarrow \frac{-\mu_1}{R} + \frac{\mu_1}{v} = \frac{-\mu_2}{R} + \frac{\mu_2}{v}$

$\Rightarrow \boxed{\frac{-\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$

* DENSER TO RARER [VIRTUAL IMAGE]



$\mu_2 > \mu_1$

From ΔAOC,
$i = \alpha + \gamma$ — ①

In ΔAOI,
$r = \beta + \gamma$

Apply Snell's law at point A,
$\mu_2 \sin i = \mu_1 \sin r$ — ③

If angles are small,
$\sin i \approx i$, $\sin r \approx r$
$\tan i \approx i$, $\tan r \approx r$

③ $\Rightarrow \mu_2 i = \mu_1 r$

$\Rightarrow \mu_2(\alpha+\gamma) = \mu_1(\beta+\gamma)$ [From ① and ②]

$\Rightarrow \mu_2(\tan\alpha + \tan\gamma) = \mu_1(\tan\beta + \tan\gamma)$

$\Rightarrow \mu_2\left(\frac{AM}{OM} + \frac{AM}{MC}\right) = \mu_1\left(\frac{AM}{MI} + \frac{AM}{MC}\right)$

$\Rightarrow \mu_2\left(\frac{1}{-u} + \frac{1}{R}\right) = \mu_1\left(\frac{1}{-v} + \frac{1}{R}\right)$ [$\because$ PM $\Rightarrow$ small distance; $OP \simeq OM = -u$; $IP \simeq MI = -v$; $MP \simeq MC = R$]

$\Rightarrow \frac{\mu_2}{-u} + \frac{\mu_2}{R} = \frac{\mu_1}{-v} + \frac{\mu_1}{R}$

$\Rightarrow \boxed{\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}}$

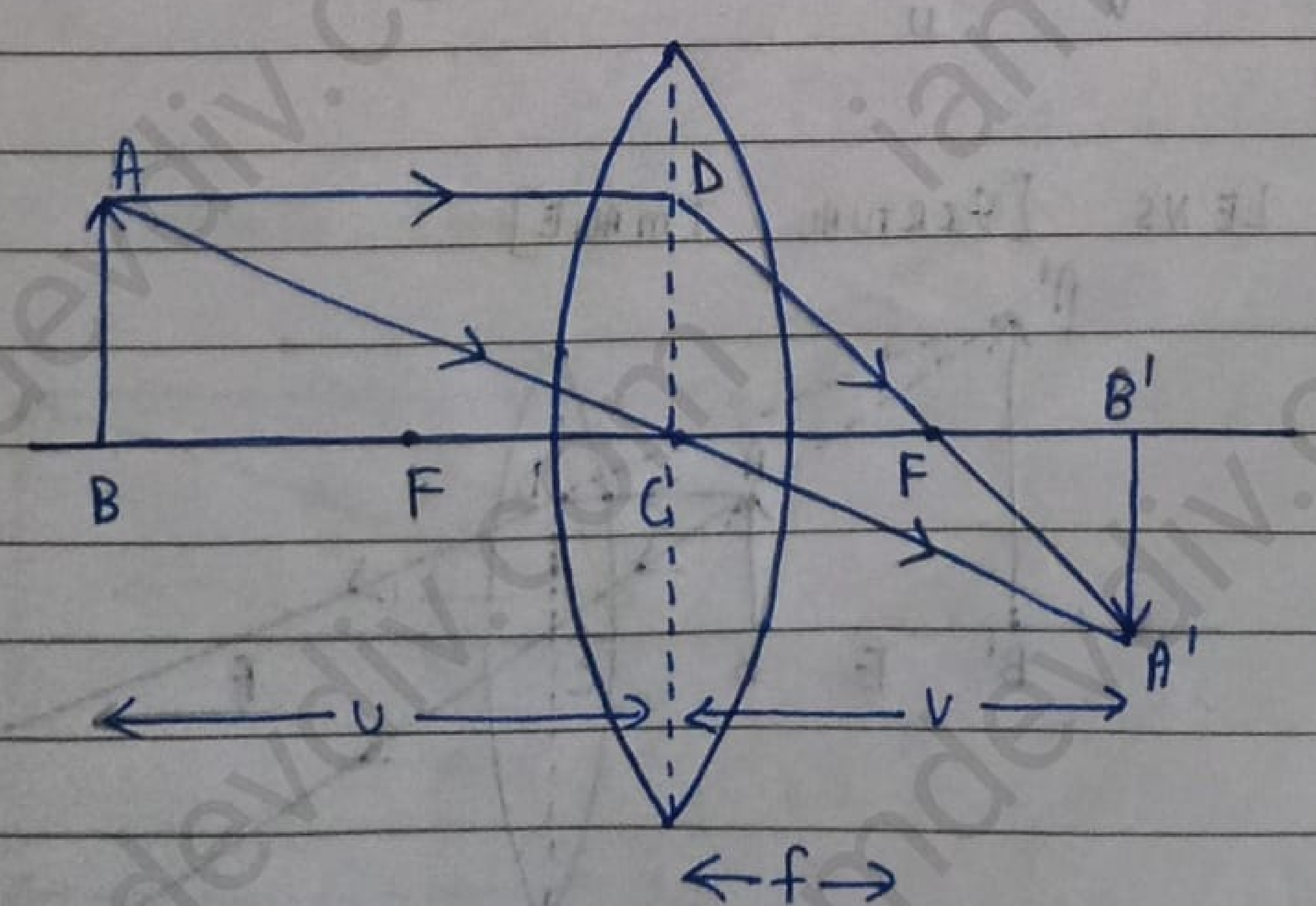
☆ LENS

A transparent medium bounded by two surface for which one or both surfaces are spherical.

(i) Convex → Converging lens (Thicker at the centre ~~bottom~~, thinner at ends)
(ii) Concave → Diverging Lens (Thicker at ends, thinner at centre)

☆ LENS FORMULAS

• CONVEX LENS [REAL IMAGE]



ΔABC and $\Delta A'B'C$ are similar

$$\frac{AB}{A'B'} = \frac{BC}{B'C} \quad \text{——— (1)}$$

ΔCDF and $\Delta A'B'F$ are similar

$$\frac{CD}{A'B'} = \frac{CF}{B'F}$$

$$\because AB = CD$$

$$\Rightarrow \frac{AB}{A'B'} = \frac{CF}{B'F} \quad \text{——— (2)}$$

From (1) and (2),

$$\frac{BC}{B'C} = \frac{CF}{B'F} = \frac{CF}{B'C - CF}$$

$$\Rightarrow \frac{-u}{v} = \frac{f}{v-f}$$

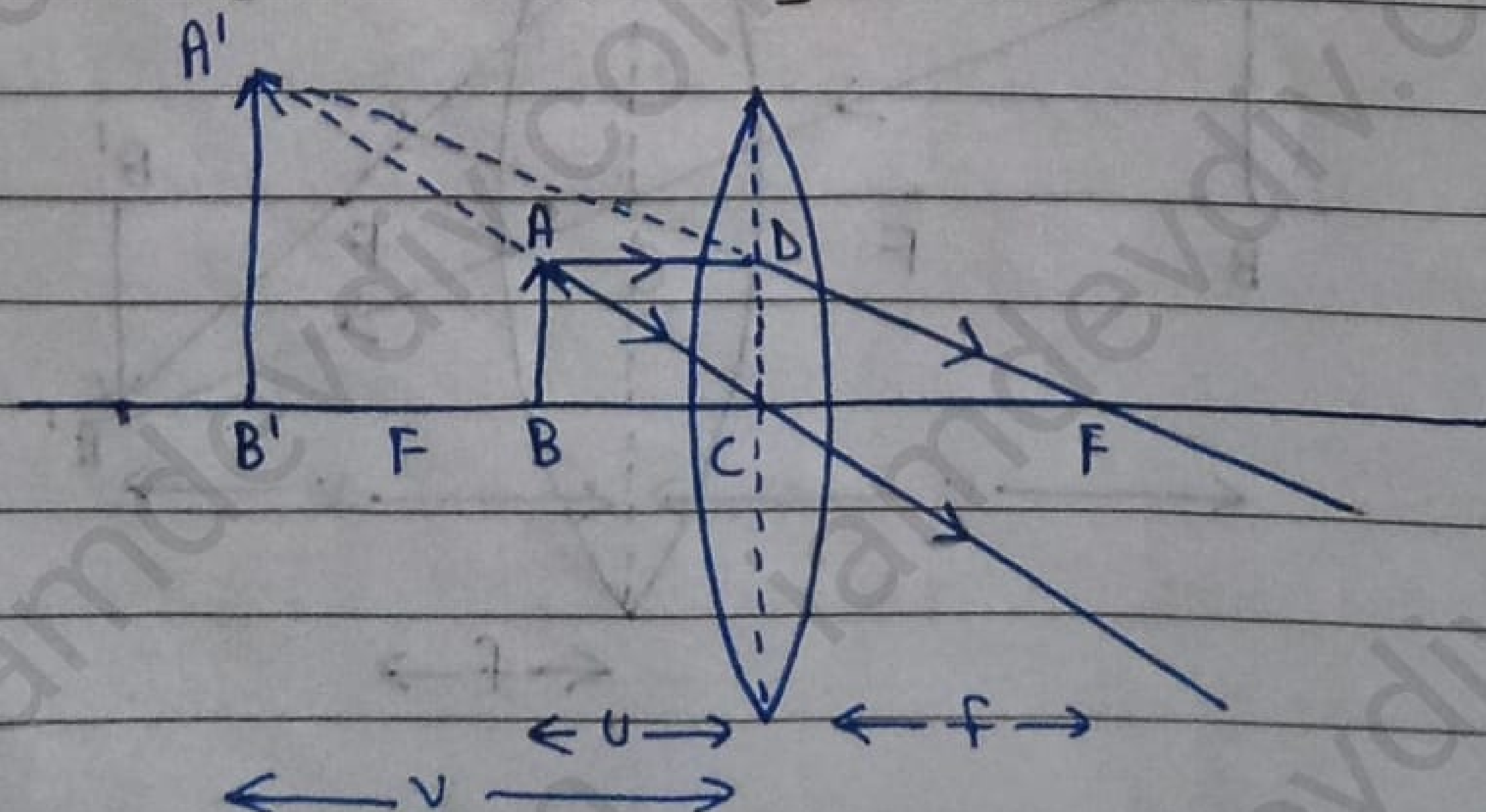
$$\Rightarrow -uv + uf = vf$$

Dividing by uvf

$$\Rightarrow -\frac{1}{f} + \frac{1}{v} = \frac{1}{u}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

- CONVEX LENS [VIRTUAL IMAGE]



$\Delta A'B'C$ and ΔABC are similar

$$\frac{A'B'}{B'C} = \frac{AB}{BC} \quad \cancel{\Rightarrow \frac{A'B'}{AB} = \frac{B'C}{BC}} \Rightarrow \frac{AB}{A'B'} = \frac{BC}{B'C}$$

ΔCDF and $\Delta A'B'F$ are similar

$$\frac{CD}{A'B'} = \frac{CF}{B'F}$$

$$\because AB = CD$$

$$\Rightarrow \frac{AB}{A'B'} = \frac{CF}{B'F} \quad ②$$

From ① and ②,

$$\frac{BC}{B'C} = \frac{CF}{B'F} = \frac{CF}{CB' + CF}$$

$$\Rightarrow \frac{-u}{-v} = \frac{f}{-v+f}$$

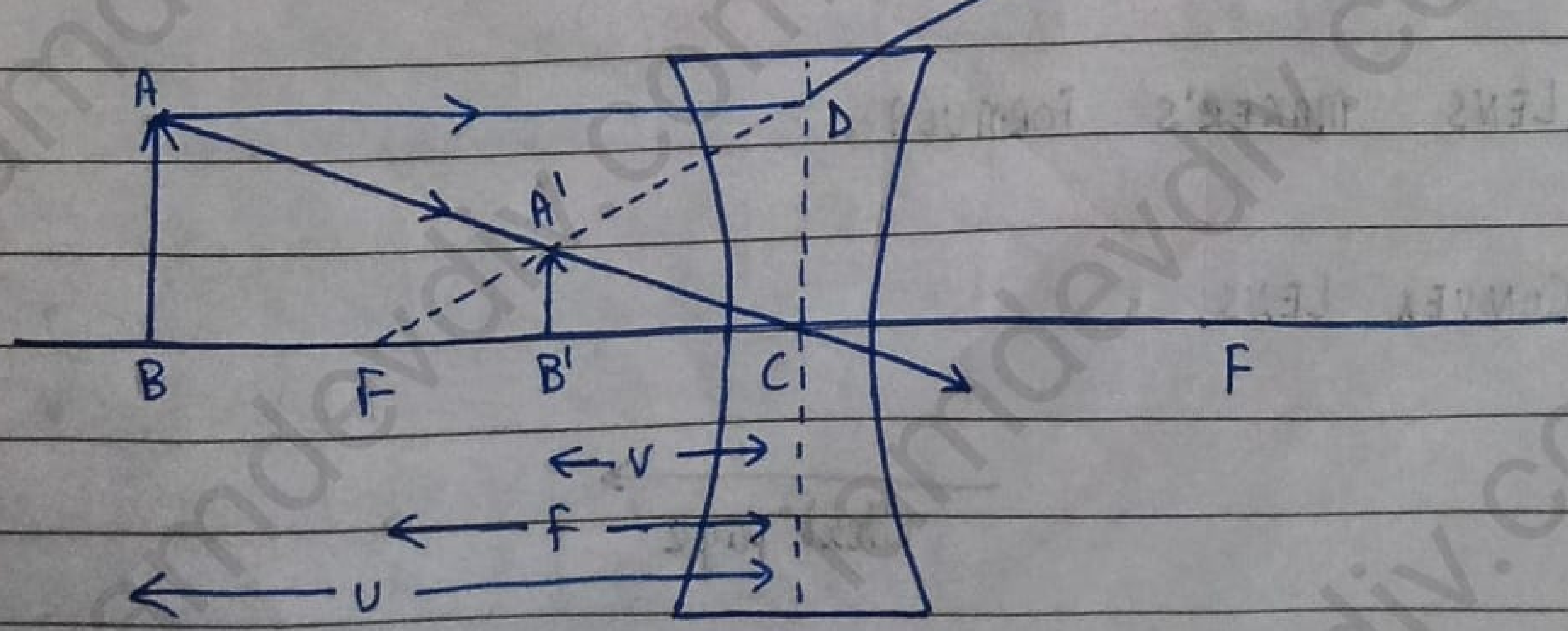
$$\Rightarrow vf = -uv + uf$$

Dividing by uvf

$$\Rightarrow \frac{1}{u} = \frac{-1}{f} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

- CONCAVE ~~CONVEX~~ LENS [VIRTUAL IMAGE]



ΔABC and $\Delta A'B'C$ are similar

$$\frac{AB}{A'B'} = \frac{BC}{B'C} \quad \text{--- (1)}$$

ΔDCF and $\Delta A'B'F$ are similar

$$\frac{DC}{A'B'} = \frac{FC}{FB'}$$

$$\because DC = AB$$

$$\Rightarrow \frac{AB}{A'B'} = \frac{FC}{FB'} \quad \text{--- (2)}$$

From (1) and (2),

$$\frac{BC}{B'C} = \frac{FC}{FB'} = \frac{FC}{FC - B'C}$$

$$\Rightarrow \frac{-u}{-v} = \frac{-f}{-f - (-v)}$$

$$\Rightarrow \frac{u}{v} = \frac{-f}{v - f}$$

$$\Rightarrow uv - uf = -vf$$

Dividing by uvf

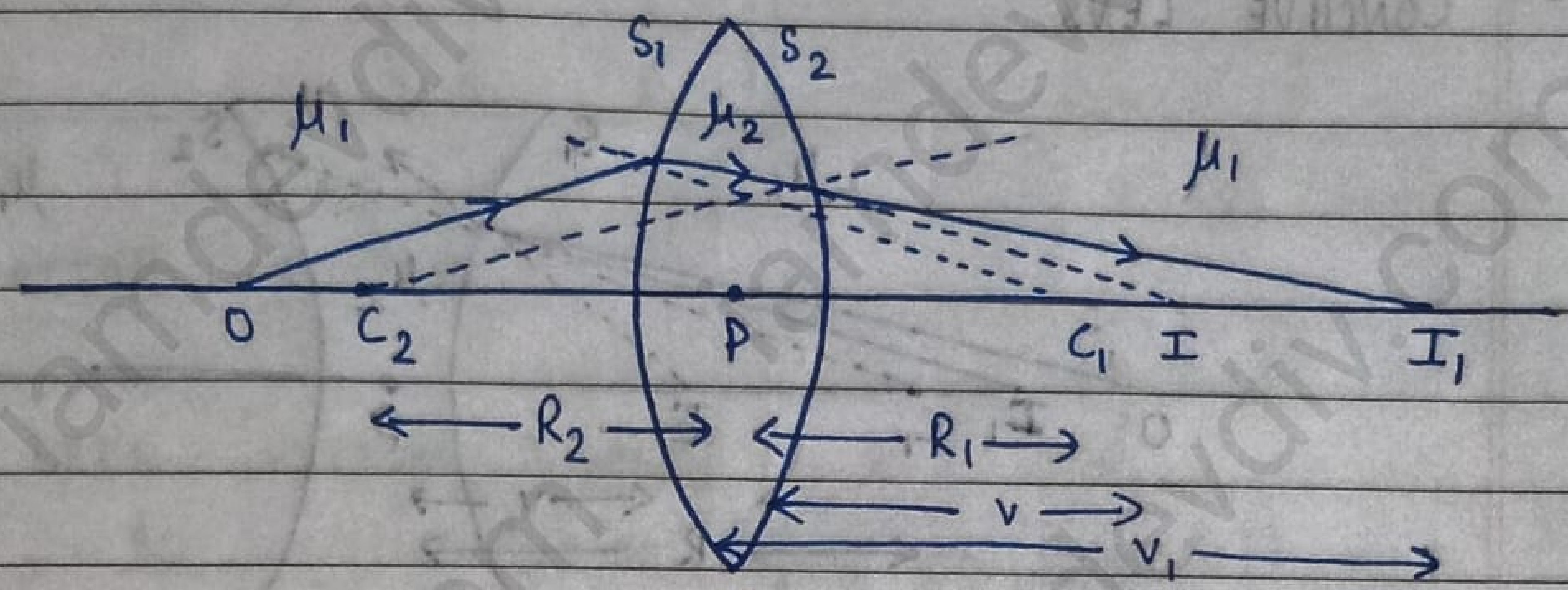
$$\Rightarrow \frac{1}{f} - \frac{1}{v} = -\frac{1}{u}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

☆ LENS MAKER'S FORMULA

• CONVEX LENS

Next page



1) Refraction at spherical convex surface S_1 (rarer to denser)

$$\frac{-\mu_1}{u} + \frac{\mu_2}{v} = \frac{1}{R}(\mu_2 - \mu_1)$$

$$\Rightarrow \frac{-\mu_1}{u} + \frac{\mu_2}{v_1} = \frac{1}{R_1}(\mu_2 - \mu_1) \quad \text{--- (1)}$$

2) Refraction at spherical convex surface S_2 (denser to rarer)

$$\frac{-\mu_2}{u} + \frac{\mu_1}{v} = \frac{1}{R}(\mu_1 - \mu_2)$$

$$\Rightarrow \frac{-\mu_2}{v_1} + \frac{\mu_1}{v} = \frac{1}{R_2}(\mu_1 - \mu_2)$$

$$\Rightarrow \frac{\mu_2}{v_1} - \frac{\mu_1}{v} = \frac{1}{R_2}(\mu_2 - \mu_1) \quad \text{--- (2)}$$

(1) − (2),

$$\frac{-\mu_1}{u} + \cancel{\frac{\mu_2}{v_1}} - \cancel{\frac{\mu_2}{v_1}} + \frac{\mu_1}{v} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

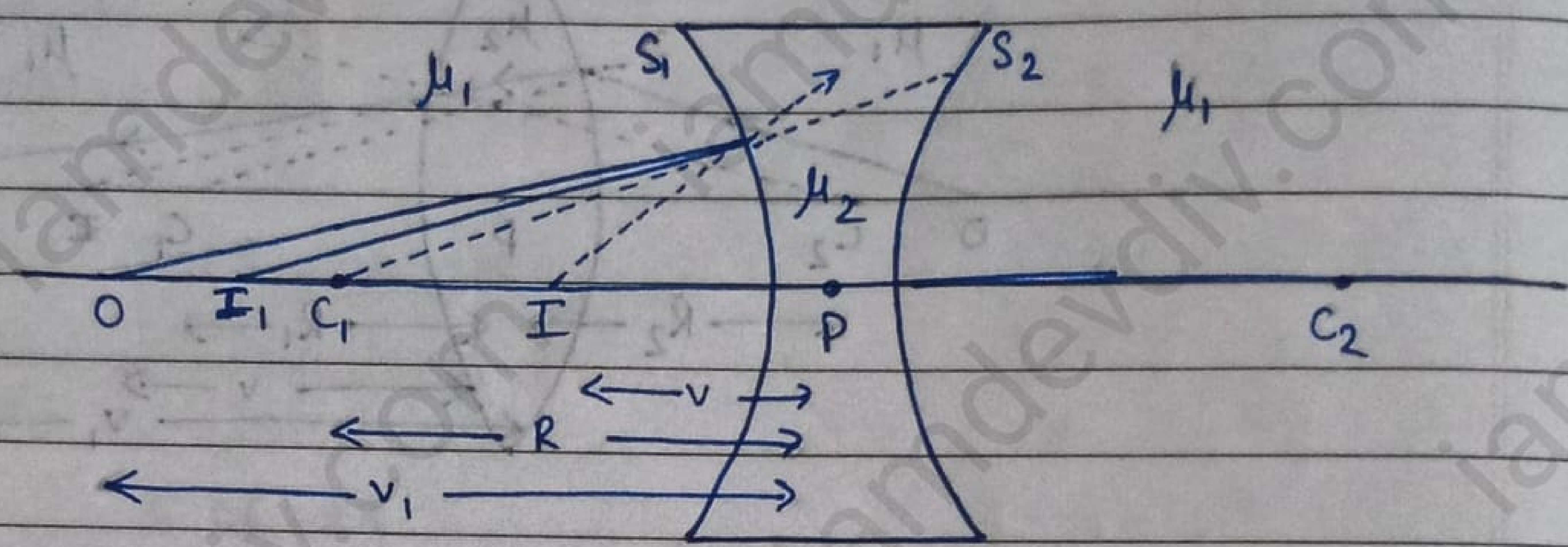
$$\Rightarrow \frac{-\mu_1}{u} + \frac{\mu_1}{v} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow \mu_1\left(\frac{1}{v} - \frac{1}{u}\right) = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - \frac{\mu_1}{\mu_2}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \boxed{\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

- CONCAVE LENS



1) Refraction at spherical concave surface S_1 (rarer to denser)

$$\frac{-\mu_1}{u} + \frac{\mu_2}{v} = \frac{1}{R}(\mu_2 - \mu_1)$$

$$\Rightarrow \frac{-\mu_1}{u} + \frac{\mu_2}{v_1} = \frac{1}{R_1}(\mu_2 - \mu_1) \quad \text{——— ①}$$

2) Refraction at spherical concave surface (denser to rarer)

$$\frac{-\mu_2}{u} + \frac{\mu_1}{v} = \frac{1}{R}(\mu_1 - \mu_2)$$

$$\Rightarrow \frac{-\mu_2}{v_1} + \frac{\mu_1}{v} = \frac{1}{R_2}(\mu_1 - \mu_2)$$

$$\Rightarrow \frac{-\mu_2}{v_1} + \frac{\mu_1}{v} = \frac{-1}{R_2}(\mu_2 - \mu_1) \quad \text{——— ②}$$

① + ②,

$$\frac{\mu_1}{u} + \cancel{\frac{\mu_2}{v_1}} - \cancel{\frac{\mu_2}{v_1}} + \frac{\mu_1}{v} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \mu_1\left(\frac{1}{v} - \frac{1}{u}\right) = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{\mu_2}{\mu_1} - \frac{\mu_1}{\mu_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \boxed{\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

☆ MAGNIFICATION OF LENS

- CONCAVE LENS [VIRTUAL IMAGE]

$$\frac{AB}{A'B'} = \frac{BC}{B'C}$$ (refer figure on page 147)

$$\Rightarrow \frac{h}{h'} = \frac{-u}{-v}$$

$$m \Rightarrow \frac{h'}{h} = \frac{v}{u}$$

- CONVEX LENS [VIRTUAL IMAGE]

$$\frac{AB}{A'B'} = \frac{BC}{B'C}$$ (refer figure on page 146)

$$\Rightarrow \frac{h}{h'} = \frac{-u}{-v}$$

$$m \Rightarrow \frac{h'}{h} = \frac{v}{u}$$

- CONVEX LENS [REAL IMAGE]

$$\frac{AB}{A'B'} = \frac{BC}{B'C}$$ (refer figure on page 145)

$$\Rightarrow \frac{h}{-h'} = \frac{-u}{v}$$

$$m \Rightarrow \frac{h'}{h} = \frac{v}{u}$$

☆ POWER OF LENS

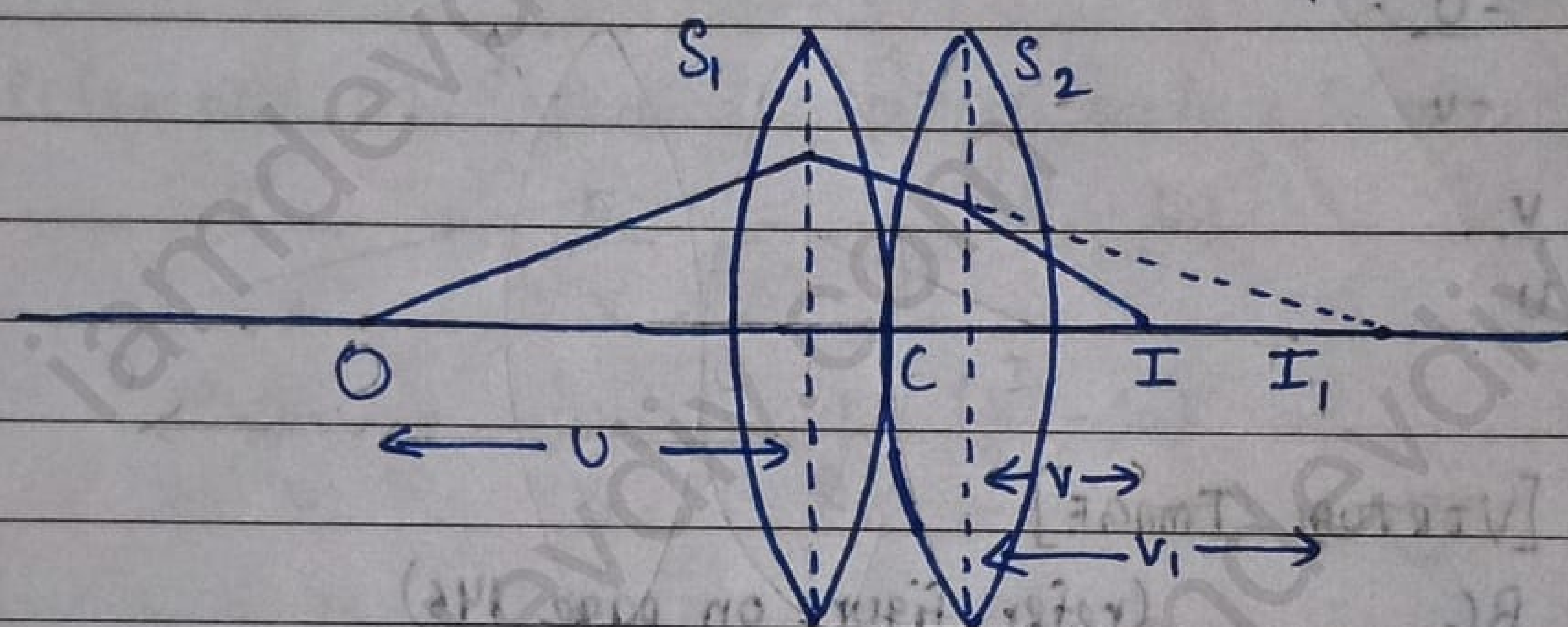
Ability of the lens to converge or diverge ray

$$P = \frac{1}{f \text{ (in metres)}}$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

☆ COMBINATION OF THIN LENSES



For S_1,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u} \quad \text{—— (1)}$$

For S_2,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f_2} = \frac{1}{v} - \frac{1}{v_1} \quad \text{—— (2)}$$

(1) + (2),

$$\frac{1}{f_1} + \frac{1}{f_2} = \cancel{\frac{1}{v_1}} - \frac{1}{u} + \frac{1}{v} - \cancel{\frac{1}{v_1}}$$

$$\Rightarrow \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F} \quad \text{—— (3)}$$

$$\Rightarrow \frac{f_1 + f_2}{f_1 f_2} = \frac{1}{F}$$

$$\Rightarrow \boxed{F = \frac{f_1 f_2}{f_1 + f_2}}$$

$$\boxed{P = P_1 + P_2}$$

Q. A double convex lens made of glass having refractive index 1.56 and has both radii of curvature of magnitude 20 cm. If an object is placed at a distance of 10 cm from the lens. Find the position of image.

Sol. $\mu = 1.56$

$R_1 = 20\ cm$, $R_2 = -20\ cm$

$u = -10\ cm$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f} = (1.56 - 1)\left(\frac{1}{20} + \frac{1}{20}\right)$$

$$\Rightarrow \frac{1}{f} = 0.56 \times \frac{1}{10}$$

$$\Rightarrow \frac{1}{f} = 0.056$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow 0.056 = \frac{1}{v} + \frac{1}{10}$$

$$\Rightarrow 0.056 - 0.1 = \frac{1}{v}$$

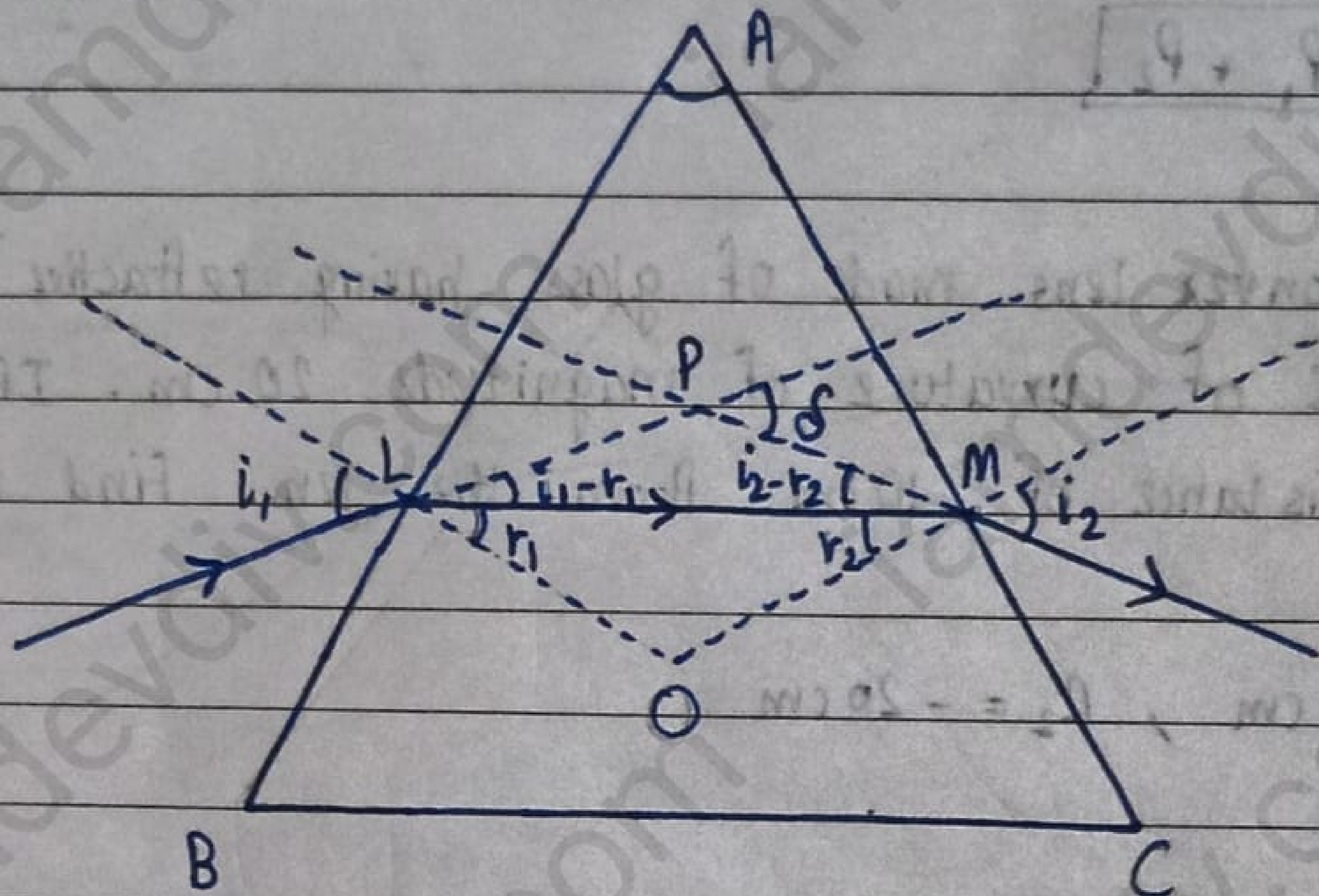
$$\Rightarrow \frac{1}{v} = -0.044$$

$$\Rightarrow v = -\frac{1000}{0.044}$$

$$\Rightarrow v = -22.7\ cm$$ Ans

★ REFRACTION THROUGH A PRISM

• CALCULATION OF ANGLE OF DEVIATION [δ]



In ΔPLM,

$$\delta = i_1 - r_1 + i_2 - r_2$$

$$\delta = i_1 + i_2 - (r_1 + r_2) \quad \text{—— ①}$$

In ΔLMO,

$$\angle r_1 + \angle r_2 + \angle O = 180^\circ \quad \text{—— ②}$$

In quadrilateral ~~ALMOA~~ ALOM

$$\angle A + \angle L + \angle M + \angle O = 360^\circ$$

$$\Rightarrow \angle A + 90^\circ + 90^\circ + \angle O = 360^\circ$$

$$\Rightarrow \angle A + \angle O = 180^\circ \quad \text{—— ③}$$

Comparing ② and ③

$$\angle A = \angle r_1 + \angle r_2$$

$$\Rightarrow A = r_1 + r_2$$

Put this value in ①

$$\boxed{\delta = i_1 + i_2 - A} \quad \text{—— ④}$$

For thin prism,

$$\mu = \frac{\sin i}{\sin r}$$

Small angles ⇒ $\sin i = i$, $\sin r = r$

$$\mu = \frac{i}{r}$$

$$i_1 = \mu r_1$$

$$i_2 = \mu r_2$$

Put these values in ④

$$\delta = \mu r_1 + \mu r_2 - A$$

$$\Rightarrow \delta = \mu(r_1 + r_2) - A$$

$$\Rightarrow \delta = \mu A - A$$

$$\Rightarrow \boxed{\delta = A(\mu - 1)}$$

- ANGLE OF MINIMUM DEVIATION AND PRISM FORMULA

$$\delta = i_1 + i_2 - A$$

$$\delta = \sqrt{i_1^2} + \sqrt{i_2^2} - A + 2\sqrt{i_1 i_2} - 2\sqrt{i_1 i_2}$$

$$\delta = \sqrt{(i_1 - i_2)^2} - A + 2\sqrt{i_1 i_2}$$

For minimum deviation,

$$\sqrt{(i_1 - i_2)^2} = 0$$

$$\boxed{\cancel{\Rightarrow i_1^2 = i_2^2}}$$

$$\Rightarrow i_1 = i_2 = i$$

~~Sin~~ $r_1 = r_2 = r$

We know that,

$$A = r_1 + r_2$$

$$\Rightarrow A = 2r$$

$$\Rightarrow r = \frac{A}{2}$$

$$\delta_m = i + i - A$$

$$\Rightarrow \delta_m = 2i - A$$

$$\Rightarrow i = \frac{A + \delta_m}{2}$$

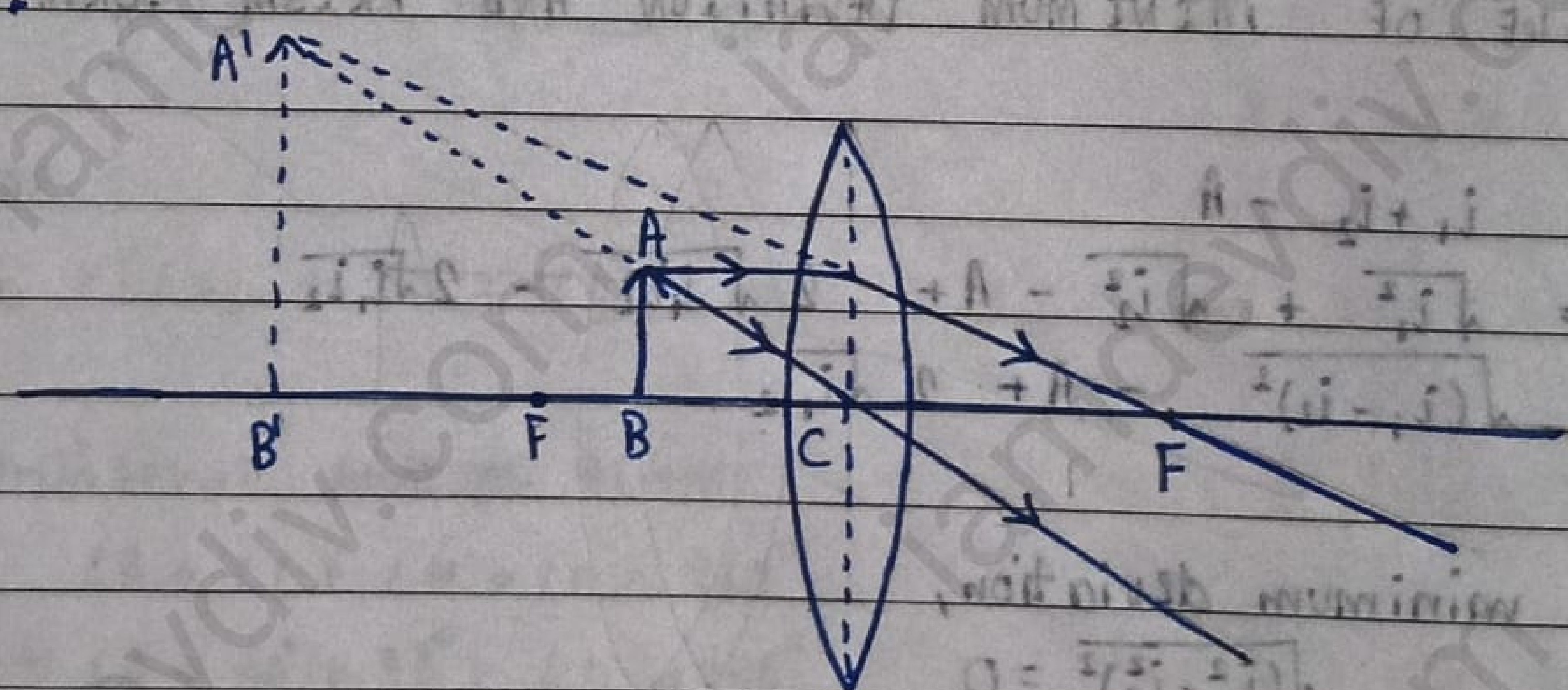
Using Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

☆ SIMPLE MICROSCOPE

It is an instrument which is used to see large image of small object.



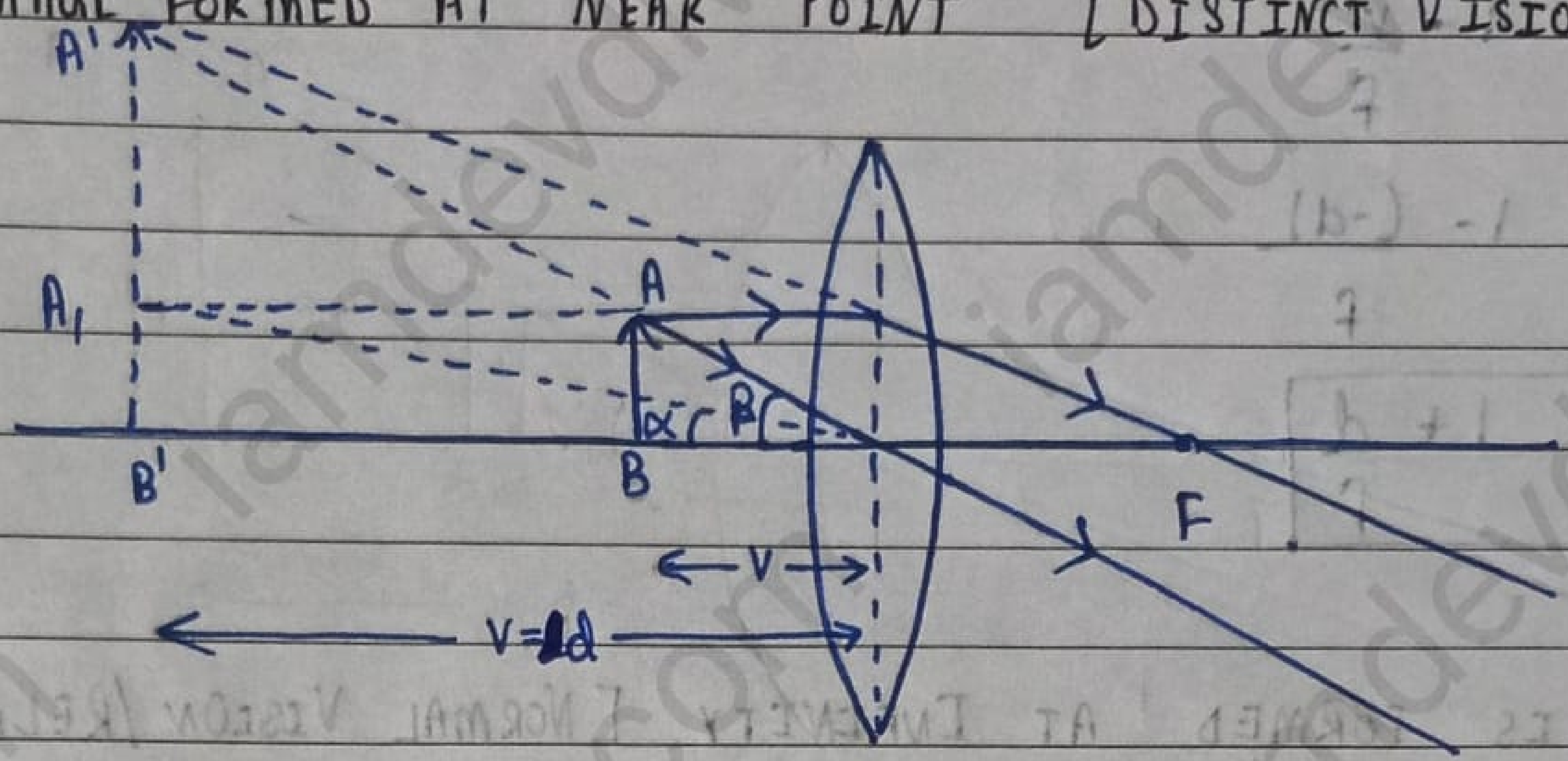
- **THEORY**

A simple microscope consists of a convex lens and the object is placed between the focus and centre of the lens. A magnified and virtual image of object is formed.

- **ANGULAR MAGNIFICATION / MAGNIFICATION**

It is defined as ~~the~~ the ratio of angle subtended by image to the angle subtended by the object on eye.

* IMAGE FORMED AT NEAR POINT [DISTINCT VISION]



$$m = \frac{\beta}{\alpha}$$

For small angles,

$$m = \frac{\tan\beta}{\tan\alpha} = \frac{A'B'/B'C}{AB'/B'C}$$

$$\Rightarrow m = \frac{A'B'}{AB'} = \frac{\text{size of image}}{\text{size of object}} \quad \text{——— (1)}$$

ΔABC and $\Delta A'B'C$ are similar,

$$\frac{A'B'}{AB} = \frac{B'C}{BC}$$

from (1),

$$m = \frac{-v}{-u} = \frac{v}{u}$$

From lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Multiplying by v,

$$\frac{v}{f} = 1 - \frac{v}{u}$$

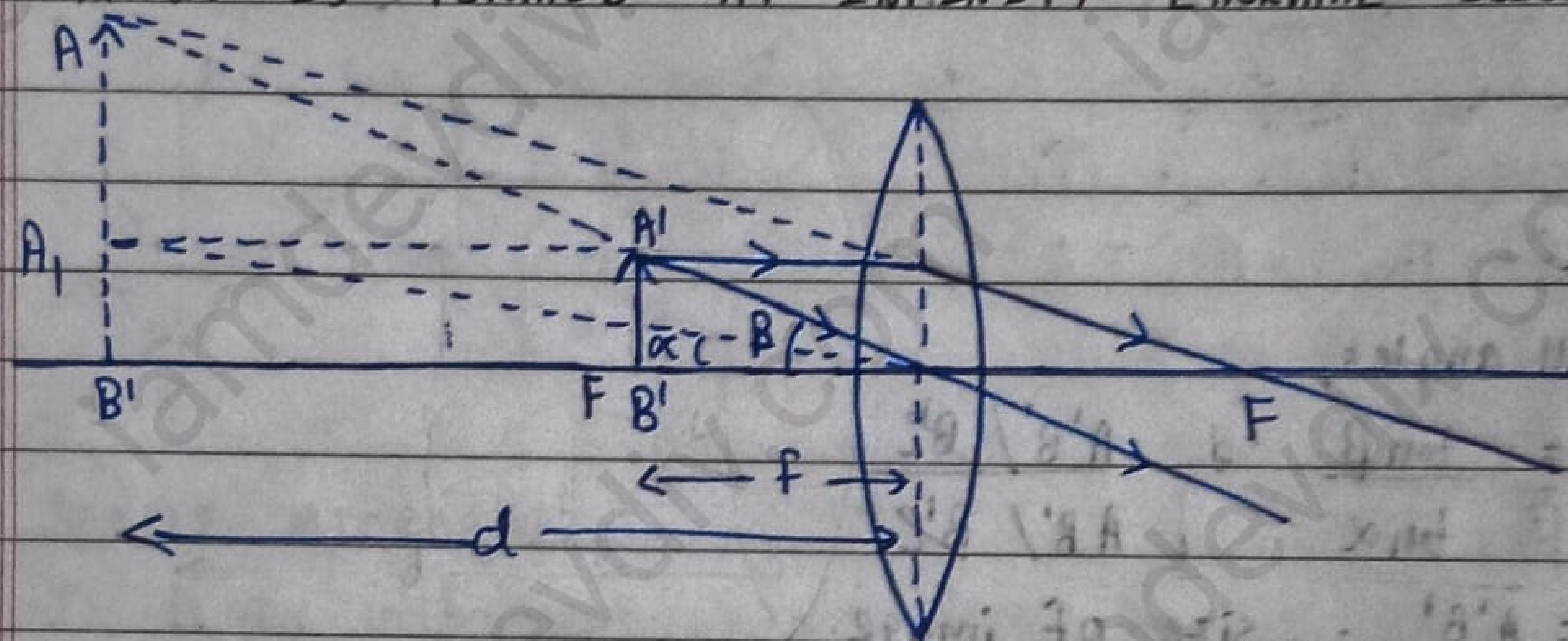
$$\Rightarrow \frac{v}{f} = 1 - m$$

$$\Rightarrow m = 1 - \frac{v}{f}$$

$$\Rightarrow m = 1 - \frac{(-d)}{f}$$

$$\Rightarrow \boxed{m = 1 + \frac{d}{f}}$$

* IMAGE IS FORMED AT INFINITY [NORMAL VISION / RELAXED EYE]

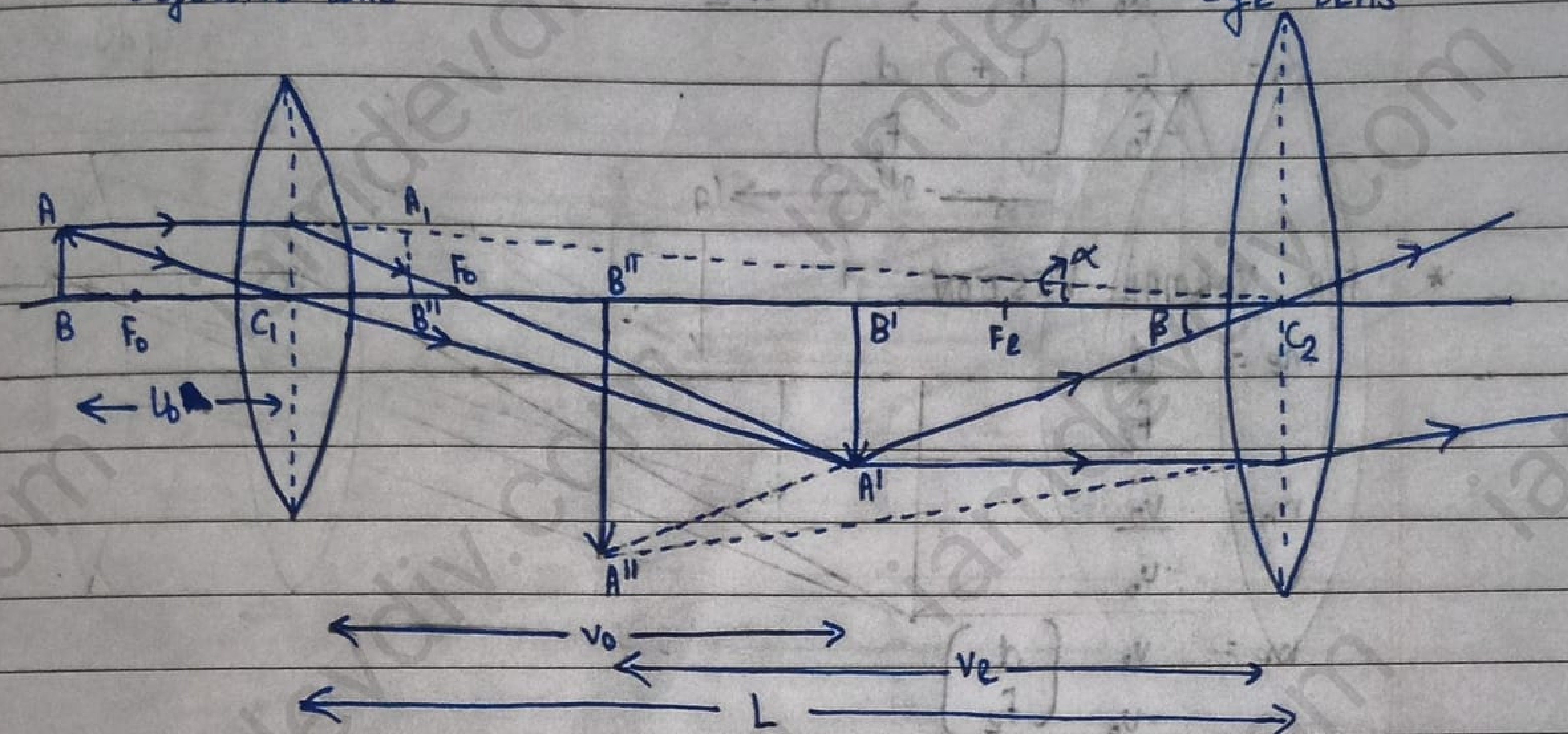


$$m = \frac{\tan\beta}{\tan\alpha}$$

$$m = \frac{\frac{AB}{f}}{\frac{A_1B}{d}} = \frac{\cancel{AB} \times B'}{f \times \cancel{AB}} = \frac{d}{f}$$

☆ COMPOUND MICROSOPE

It is a device which is used to see ~~many~~ very minute particles



$$m = \frac{\tan\beta}{\tan\alpha} = \frac{\frac{A''B''}{B''C_2}}{\frac{A_1B''}{B''C_2}} = \frac{A''B''}{AB} = \frac{\text{size of final image}}{\text{size of object}}$$

$$m = \frac{A''B''}{AB} \times \frac{A'B'}{A'B'}$$

$$\Rightarrow m = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB} = m_e \times m_o$$

* FOR DISTINCT VISION

$$m = m_e \times m_o$$

$$m_e = 1 + \frac{d}{f_e}$$

$$m_o = \frac{v_o}{-u_o}$$

$$m = m_o \times m_e$$

$$= \frac{v_o}{-u_o}\left(1 + \frac{d}{f_e}\right)$$

- $u_0 \simeq f_0 \quad , \quad v_0 \simeq L$

$$m = \frac{L}{-f_0}\left(1 + \frac{d}{f_e}\right)$$

* FOR NORMAL VISION

$$m_e = \frac{d}{f_e}$$

$$m_0 = \frac{v_0}{-u_0}$$

$$m = \frac{v_0}{-u_0}\left(\frac{d}{f_e}\right)$$

$u_0 \simeq f_0 \quad , \quad v_0 \simeq L$

$$\Rightarrow m = \frac{L}{-f_0}\left(\frac{d}{f_e}\right)$$

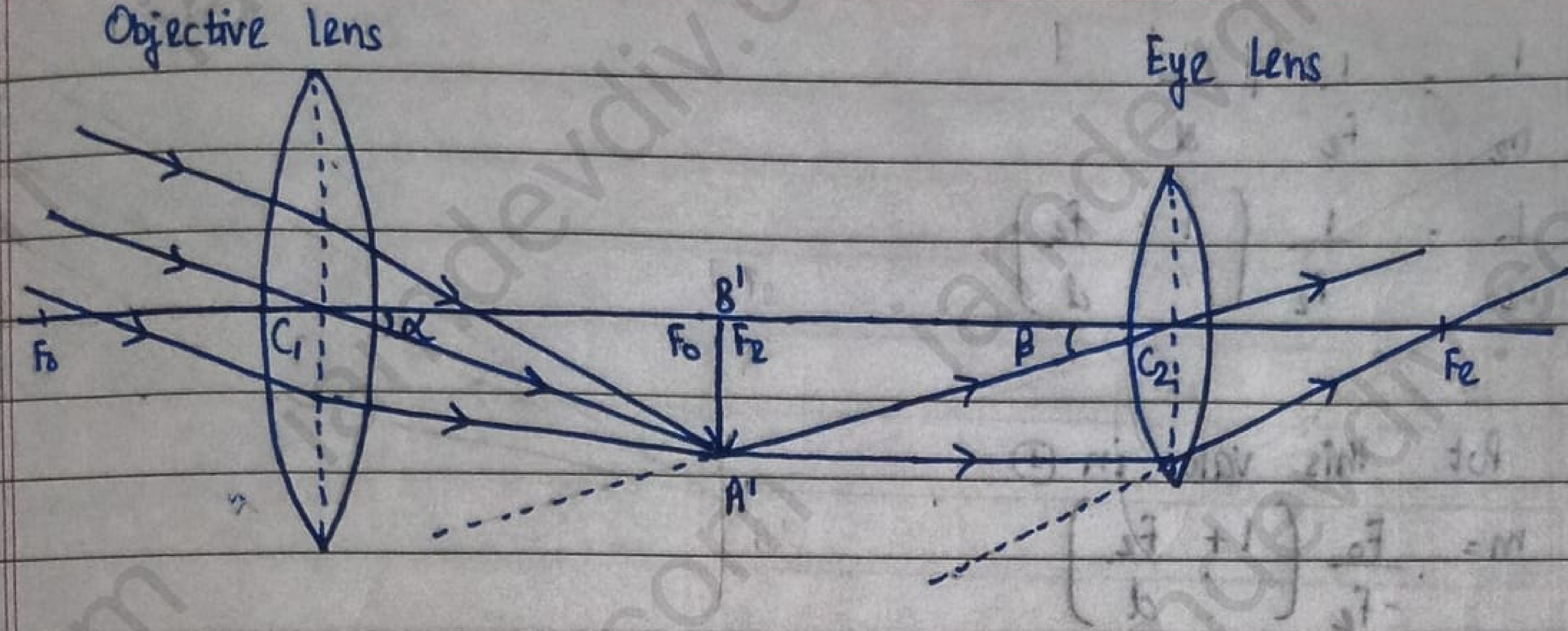
☆ TELESCOPE

It is a device which is used for observing distinct images of heavy body like stars and planets.

- THEORY

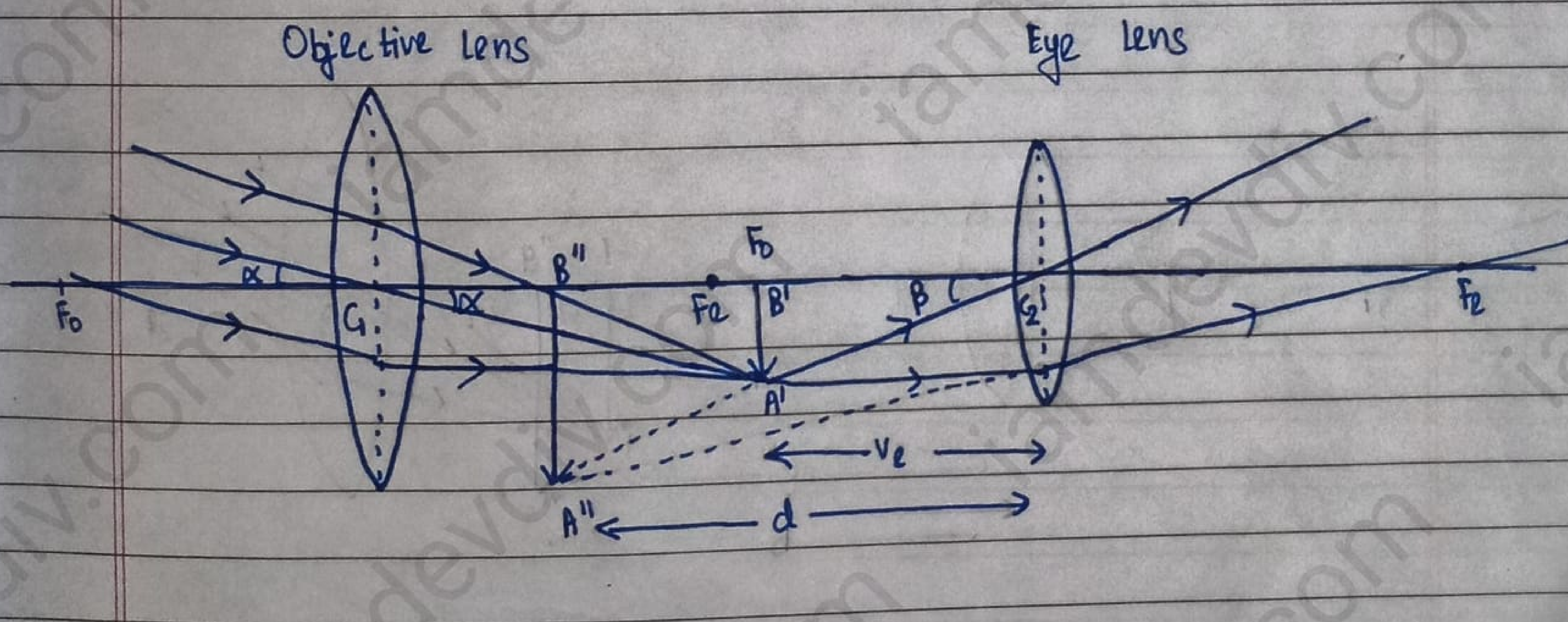
It has two convex lens separated by same distance. The lens towards the object is called objective lens. The lens towards the eye is called eye lens. Objective lens has large aperture than eye lens.

* IMAGE IS FORMED AT INFINITY



$$m = \frac{\tan\beta}{\tan\alpha} = \frac{\frac{A'B'}{B'C_2}}{\frac{A'B'}{B'C_1}} = \frac{B'C_1}{B'C_2} = \frac{f_o}{-f_e}$$

* IMAGE IS FORMED AT NEAR POINT



$$m = \frac{\tan\beta}{\tan\alpha} = \frac{\frac{A'B'}{B'C_2}}{\frac{A'B'}{B'C_1}} = \frac{B'C_1}{B'C_2} = \frac{f_o}{-v_e} \quad (1)$$

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

$$\Rightarrow \frac{1}{f_e} = -\frac{1}{d} + \frac{1}{u_e}$$

$$\Rightarrow \frac{1}{v_e} = \frac{1}{f_e} + \frac{1}{d}$$

$$\Rightarrow \frac{1}{v_e} = \frac{1}{f_e}\left[1 + \frac{f_e}{d}\right]$$

Put this value in ①

$$m = \frac{f_o}{-f_e}\left(1 + \frac{f_e}{d}\right)$$